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AN ANALYSIS OF THE DEMAND DISTRIBUTIONS FOR SELECTED INERTIAL MEASUREMENT UNITS IN THE AIR FORCE INVENTORY

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LSSR 5-78A

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Within the Air Force Logistics Command's Recoverable Item Requirements Computation System (D041), a simple Poisson demand distribution is assumed in order to forecast requirements and, thereby, to establish inventory levels for recoverable items. This study is pursuant to the question of the validity of this assumption. Detailed analysis of failure data for the FLIP, LN-15, and KT-73 inertial measurement units revealed that the demand distribution for these items was not Poisson, but more generally, negative binomial. Use of a negative binomial demand distribution in the Variable Safety Level (VSL) portion of D041, however, requires that the steady state probabilities for that demand distribution be computed. Thus far, this computation has proven to be intractable. Further research uncovered the possibility that a compounded form of the Poisson distribution might also describe the failure data, but research in this area was deferred because SIMFIT, the data analysis program used in the study, did not include subroutines to examine this sort of distribution.

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Presented to the Faculty of the School of Systems and Logistics

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In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics Management

By

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CHAPTER I

INTRODUCTION

Logistical support of an operational weapon system requires that spares be available to replace components that fail. The cost of the complex equipment used in modern weapon systems dictates that the number of spares in the logistics system be kept to a minimum. Determination of minimum spares requirements depends upon an accurate prediction of expected failures.

Background

The Air Force distinguishes between initial and replenishment spares. Usually, initial spares support a weapon system from the time of its preliminary operational capability through the item lead time plus three months. In no case, is the initial support period less than twelve months (2:p.1-1). Replenishment spares support the weapon system thereafter.

Spares are classified as either recoverable or expense items. Recoverable items are repaired and returned to service if they fail; expense items are thrown away. Expense item replenishment requirements are computed under Economic Order Quantity procedures (5). More complex procedures for computing recoverable item

replenishment requirements are embodied in the Recoverable Consumption Item Requirements System, designated D041 (3:p.1-1).

The D041 System. The D041 system operates at each of the Air Force Logistics Command's five Air Logistics Centers. A segment of the system also operates at HQ AFLC. D041 is designed to accomplish the following:

a. Compute requirements for recoverable . . . items . . . .
b. Perform the routine clerical, mathematical, and statistical workload involved in computing recoverable item requirements.
c. Forecast gross and net requirements using past and future programs, usage history, and asset information maintained within this system.
d. Produce reports for management evaluation and action.
e. Produce information for other automated data systems [3:p.2-1].

Recoverable item requirements are computed by the D041 system once every quarter of the fiscal year. The first computation of the fiscal year is used to develop apportionment requests and the third, to develop budget requests. All four computations are used to identify items requiring logistics actions, such as buy, repair, disposal, or termination of existing procurement actions (3:pp.1-1 to 1-2).

The D041 system incorporates a complex requirements computation algorithm. The algorithm first multiplies a forecast of future program activity by a series of factors to generate a gross requirement. The gross requirement

consists of an operating requirement, a base stock level, a depot stock level, plus several miscellaneous categories of requirements that have no bearing on the subject of this research. The next step is to apply to the gross requirement those assets forecast to be available. In so doing, a forecast is made of the base and depot level repair to be accomplished in the computation period. Requirements that cannot be satisfied by the inventory, with maintenance support, must be supported by additional procurement (3:Ch.6).

The D041 computation incorporates a Variable Safety Level (VSL) feature based on the METRIC (Multi-Echelon Technique for Recoverable Item Control) model developed for the Air Force by The RAND Corporation. A description of METRIC can be found in a RAND Memorandum, RM-5078-PR (21). Use of the METRIC model requires specifying the probability distribution associated with the demand process (17:7). In D041, all demands are assumed to occur according to a Poisson distribution (15). This research examined the validity of the assumption as it pertains to computing requirements for specific items.

<u>The Poisson Process</u>. A few comments on the Poisson process will provide the statistical concepts underlying the problem that this research addresses. "A large class of situations in which events occur randomly is the

Poisson process [16:374-375]." Two probability distributions, the Poisson and the exponential, are associated with a Poisson process. The probabilities for the number of times an event occurs in an interval of time follow a Poisson distribution; while the probabilities for the times between occurrences of that event follow an exponential distribution (16:375). One of the assumptions that must be met in order for a Poisson process to apply is that "the process rate λ *must remain constant* for the entire duration considered [16:376]."

Statement of the Problem

Recent research (8) by Captains Lowell R. Crowe and Levi D. Lowman, Jr., addressed the failure patterns experienced by three inertial measurement units (IMUs): the FLIP unit used on the C-5A aircraft, the LN-15 unit used on the B-52G/H aircraft, and the KT-73 unit used on the A-7D/E and AC-130H aircraft. They concluded that the *best* theoretical distribution for describing the actual operating time to failure of the IMUs was *not* the exponential (8:44-48). They also reported indications that the IMU failure rates may have changed over time (8:71).

These findings call into question whether the failures of these IMUs can be legitimately considered to be derived from a Poisson process. First, of course, the applicability of the exponential distribution itself

is questioned, although the evidence does not conclusively eliminate it from consideration. The inconclusiveness stems from the decision rule used to determine the best distribution. The best distribution was that which described the most samples from each IMU population. In fact, the exponential *did* fit some of the samples (8:50-53). Second, the assumption of a constant process rate was undermined.

Unless the failures of the three IMUs can be considered to be derived from a Poisson process, it would be inappropriate to use the Poisson distribution to model demands in the computation of requirements for these items. The problem this research addressed has three basic components. First, there was the question of whether the Poisson process does, in fact, represent the failure pattern for the three IMUs. Second, there was the question of whether substituting the empirical failure distribution for the IMUs (or another appropriate theoretical distribution) in the computation would significantly change the computed requirement. Finally, there is the possibility, if these IMUs do not exhibit Poisson characteristics, that the same would apply to other IMUs.

Approach to the Research

This research was intended to pursue the first two components of the problem just discussed. A two-phased approach was employed. In the first phase, the

question of what probability distribution fits the failure data was to be settled. If the Poisson was discredited, a second phase was to be undertaken to determine whether incorporation of an alternative distribution significantly changes the computed requirement. The final aspect of the problem would have required the collection and evaluation of so much additional failure data that it appeared reasonable not to do so unless there were indications from this research to warrant the effort.

Justification

It can be assumed that a requirements computation reflecting the actual demand distribution exprienced by an item would be more accurate than one reflecting an inappropriate distribution. A more accurate computation would have one of two results. In the first case, a smaller requirement would be computed, saving the Air Force the money it would have spent for repair or procurement. In the second, a larger requirement would be computed, necessitating more repair and procurement funds.

Improved accuracy, while desirable, is not free. In order to implement this kind of improvement, it is possible that the Air Force would have to change the data collection system that feeds the requirements process. The current system collects only the total number of failures and total operating time for a component

population (8:6). These data, as previously noted, are used in determining the demand rate. However, they are not sufficient for determining the distribution from which the demands are taken unless that distribution is defined by only one parameter. For distributions that are defined by two or more parameters, unaggregated data would have to be collected in such a way that the parameters could be estimated directly from the data.

The costs associated with providing the additional data to the requirements system are not the only ones that might be incurred. The cost of performing the computation itself could be increased. A 1970 AFLC study cited the computational efficiency of using the Poisson rather than other discrete probability distributions and noted that corrections would be required to use continuous distributions to approximate an inherently discrete process such as demand generation (22:29). Additional computational costs could, therefore, be anticipated if any probability distribution other than the Poisson were incorporated into the requirements computation algorithm.

If the requirements computation were not sensitive to the differences in demand embodied in alternative probability distributions, significantly improved accuracy would not be expected. If the requirements computation were, however, sensitive to these differences, the magnitude of the change would influence whether an improved technique

would pay for itself in savings or whether its effect would warrant the costs associated with its implementation. The Air Force Logistics Management Center, Gunter Air Force Station, Alabama, has requested that this sensitivity be investigated (23).

Objectives of the Research

This research had three objectives. Specifically, the research sought to:

1. Identify the distribution that best describes the occurrence of failures for each IMU within a time frame appropriate to the requirements computation.

2. Determine how to modify the D041 computation to incorporate the identified distribution.

3. Determine whether the computation is sensitive to the proposed modification through a comparison of requirements for three inertial measurement units computed using both the standard and the modified D041 process.

Research Hypotheses

1. For each of the inertial measurement units under study, the daily occurrences of failure do not follow a Poisson distribution.

2. Spares computations are sensitive to the probability distribution that is assumed to describe the underlying equipment failure pattern.

CHAPTER II

METHODOLOGY

This chapter presents the methodology used for identification of the distribution that best describes the occurrence of failures for each of the three IMUs. The first subject to be addressed is the organization of the data which was used. The data collection plan, including a discussion of the G078C data processing system from which the data were taken, will follow. Finally, there will be a general discussion of the analysis techniques employed.

Population Defined

Failure data for all aircraft inertial measurement units (IMUs) in the Air Force inventory constitute the universe for this research. Within the universe, three discrete populations were defined as failure data applicable to the FLIP unit used on the C-5A, the LN-15 unit used on the B-52G/H, and the KT-73 unit used on the A-7D/E and the AC-130H.

Failure data associated with each IMU population were arranged in chronological order by failure date. Population subsets were defined by partitioning the data by base period. A base period was defined as eight

consecutive calendar quarters. The eight quarter block was chosen as the basis for investigation because the requirements computation system (D041) uses eight quarters of failure history for its demand forecasts (2:p.1-4). Figure 1 illustrates the partition of failure data by base period. Base Period 1 includes the eight quarters beginning with the first one in which an IMU failure occurred (e.g, in Figure 1, quarters 70-1 through 71-4). In Base Period 2, the first quarter of the previous base period is dropped and another quarter added at the end (e.g., quarters 70-2 through 72-1). This process is repeated until the last complete eight-quarter base period has been partitioned.

Census data on the occurrences of failure for each IMU from its introduction into the inventory until 30 June 1977 were obtained. Reasons for establishing the 30 June cutoff date will be discussed within the context of the data collection plan. Failures occurring before the first quarter in which at least twenty failures are recorded were eliminated from further consideration. This criterion was arbitrarily established to minimize the effects of system startup on later analysis and to delimit the number of base periods to be investigated. Figures 2, 3, and 4 identify the data that was used in the analysis.

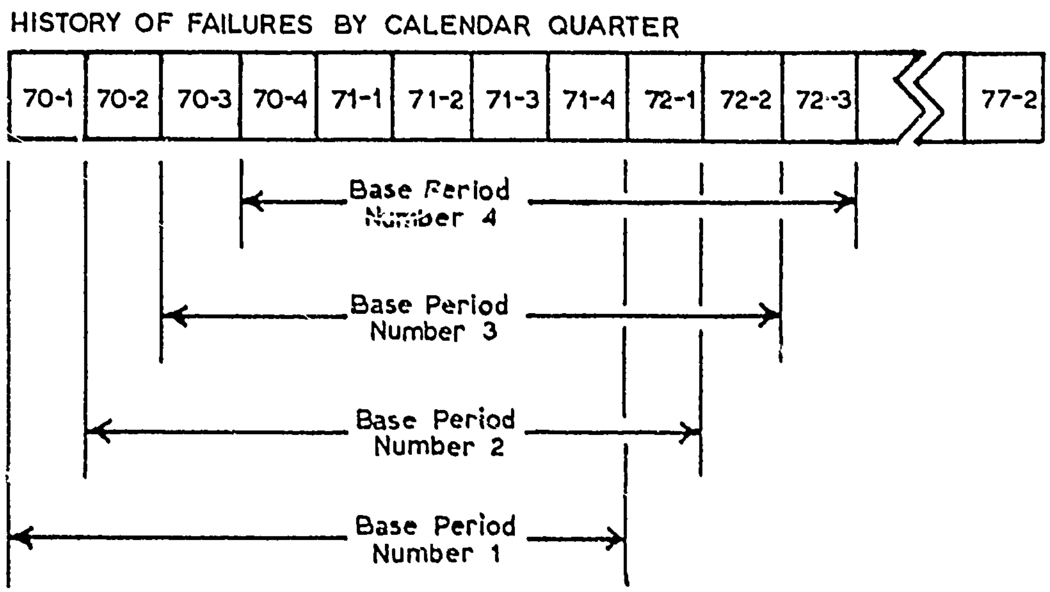

Fig. 1. Partition of Failure Data by Base Period

FLIP	LN-15	KT-73
1 July 1971 through 30 June 1977	1 January 1974 through 30 June 1977	1 January 1972 through 30 June 1977

Fig. 2. Inclusive Dates of Data for Each IMU

FLIP	LN-15	KT-73
1,108	723	2,065

Fig. 3. Number of Failures in Data Base for Each IMU

FLIP	LN-15	KT-73
16	6	14

Fig. 4. Number of Complete Base Pericds Available for Analysis for Each IMU

Data Collection Plan

Unaggregated failure data for the three IMUs are available in the automated Inertial Guidance Reliability and Configuration Program for Aircraft, designated the G078C data system. The G078C data base contains five types of records: (1) inertial system history, (2) test data, (3 and 4) subindenture records, and (5) nonserialized piece parts (4:p.2-1). This research is concerned with the data contained on the first type of record which is established each time a failed IMU is returned to the depot for repair. The data in G078C were verified by personnel of the Aerospace Guidance and Metrology Center (AGMC), Newark Air Force Station, Ohio, and are considered to be accurate by engineers of the Reliability and Operations Division (AGMC/SNO) who routinely use the data (14; 19).

Among the data recorded when a type 1 record is established is the date of the unit's arrival at the depot. Until approximately 1 July 1977, all three IMUs were managed under SCARS (Serialized Control and Reporting System) procedures. As long as SCARS procedures were in effect, the date of arrival reflected in the G078C data base represented the actual date on which the item arrived at the depot from the base at which the failure actually occurred. Since that time, the date of arrival recorded in the G078C has reflected the date on which the

unit was input to the depot work center for repair (14). The change in reporting procedures provided a logical point at which to establish the data cutoff.

It should be noted that the date of arrival at the repair depot is not the same as the date of failure. The possibility of directly obtaining the failure date was considered. The date on which the failed IMU was removed from an aircraft is reported on AFTO Form 88, Historical Record for Inertial Navigation Unit (24:p.2-4). A complete file of AFTO Forms 88 is maintained at AGMC by the Reliability and Operations Division (14). It was determined that the effort required to extract the data from the forms, validate them against the G078C data, and manually build new data files would be excessive.

As an alternative to obtaining the date of failure directly from AFTO Forms 88, the possibility of adjusting the arrival date, as recorded in G078C, to approximate the actual failure date was also considered. The SCARS Management Indicator Report (RCS:LOG-LO7(Q)71224), prepared quarterly by the SCARS monitor at the Air Logistics Center at which the IMU is managed, reflects the average number of days spent in transit from base to depot for each IMU (26:p.13-6). Adjusting all arrival dates within a quarter by the same amount would only have shifted the data and would not have yielded useful

information concerning actual failure dates. This alternative, therefore, was rejected.

Due to the fact that the actual failure dates are not readily available, it was assumed that the pattern of unserviceable arrivals at the depot accurately reflects the distribution of failures as they actually occurred.

Data Preparation

Arrival dates were extracted from the G078C type 1 records. These dates are located in columns 17 through 21 of the standard, eighty-column, single-card record (4:A5-1). Three files of data were established, one for each IMU. Each of these files was sorted chronologically in order to partition the data into base periods. Once sorted in this manner, multiple arrivals on a given date were indicated by multiple occurrences of that date on the file. Individual files for each base period were extracted from the three chronological master files.

Each of the base period files was then processed using a computer program developed by the authors. The program accomplished the following:

1. The frequency with which each date appeared on the file was recorded.

2. Dates which did not appear on the file were recorded with a frequency of zero.

3. All dates occurring on Saturday and Sunday were eliminated from the file.[1]

The resulting data files were in the format required for the analysis procedure described in the following section.

Research Design

The hypothesis that the number of arrivals per day in each base period follows a Poisson distribution was tested using SIMFIT, a computer program on the CREATE[2] system. SIMFIT, written in FORTRAN IV, uses a histogram construction technique to compare theoretical distributions to data supplied by the user.[3]

Two nonparametric Goodness-of-Fit tests, the Kolmogorov-Smirnov One Sample (K-S) and the Chi-Square (X^2), are employed in SIMFIT to determine whether a given theoretical distribution adequately describes the user's

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[1] With the exception of four arrivals which occurred on weekends in the KT-73 master file, no arrivals were found on either Saturday or Sunday in any of the data files. Since arrivals at the depot could not be expected on weekends, it did not appear reasonable to include weekend dates since they accounted for over 25 percent of the dates in a base period.

[2] CREATE is an acronym for Computer Resources for Engineering, Analysis, Training, and Education. The system is located at Wright-Patterson AFB OH and supports AFLC and Air Force Institute of Technology (AFIT) activities.

[3] Appendix A provides a brief description of the SIMFIT computer program.

data. These two particular tests were selected for inclusion in the program because of their distribution-free property. The K-S test is preferred over the χ^2 test because of its relatively greater power in determining goodness-of-fit (25:Ch.2). One limiting factor of the K-S test is that it is not applicable when the population parameters cannot be specified in advance of the test. The χ^2 test should normally be used when the population parameters must be estimated from sample data (12:86).

SIMFIT determines the mean and variance of the input data and uses them to calculate the parameter(s) of the distribution being tested unless the user specifies the parameter(s). In this research, the data for each base period represented a subset of the population rather than a sample from the population. Therefore, the mean and variance of the data were considered to be population parameters rather than estimates derived from a sample, and it was considered appropriate to use the K-S test.

In addition, SIMFIT assumes, for the purposes of the χ^2 test, the parameters are *always* estimated from a sample and reduces the degrees of freedom for the test statistics commensurately (1). For these reasons, the K-S test was used exclusively as the measure of the goodness-of-fit for this research.

The results of the K-S test were used to accept or reject a particular distribution as the underlying distribution for the arrivals in a given base period. SIMFIT offers a choice of three levels of confidence for the G-O-F tests: 90 percent, 95 percent, and 99 percent (25:2). A level of 90 percent was chosen as appropriate because it is the most conservative of the three. This means that $\alpha=.10$ offers the greatest opportunity available to reject a particular distribution for a given set of data.

The statistical hypothesis tested is stated as follows:

Null H_0: $x \sim$ the hypothesized distribution with the appropriate parameter(s).

Alternate H_1: $x \not\sim$ the hypothesized distribution with the appropriate parameter(s).

where x is the number of arrivals per day. The null hypothesis was rejected at the 90 percent level of confidence using the K-S G-O-F test if the calculated value of D (the K-S statistic) was greater than the critical value of D. The criterion for identifying the theoretical distribution that best describes the distribution of

arrivals for each IMU population was established as the distribution that passed the K-S test for the greatest number of base periods.

Research Assumptions

The following assumptions apply to this research:

1. Individual IMU failures are independent, but the rate at which failures occur may change over time.

2. The data extracted from the G078C data base are accurate.

3. SIMFIT reliably fits known distributions to sample data and accurately estimates the parameters of those distributions.

4. The distribution of unserviceable IMU arrivals at the depot accurately reflects the occurrence of IMU failures.

5. The subset of a population may be treated as a discrete population for statistical purposes.

Research Limitations

The following limitations to the research effort have been identified:

1. There has been no attempt to establish that the three IMUs studied are representative of all aircraft IMUs, therefore, the findings of this research should not be generalized without further corroboration.

2. Findings are limited to an indication of whether there is a problem, in terms of the accuracy of the computation, associated with the using of the Poisson instead of a more appropriate distribution. No attempt has been made to assess the number of items involved, nor to determine the costs associated with rectifying any problem.

CHAPTER III

ANALYSIS AND FINDINGS

The methodology for determining the distribution that best fits the failure data included in each of the base period data files was discussed in the last chapter. The approach taken was first to determine whether, in fact, the Poisson distribution passed the K-S test for any of the base periods. Once that was done, other distributions that might describe the data were examined using the SIMFIT computer program.

The Poisson Distribution

The first hypothesis addressed was that the occurrences of failure do not follow a Poisson distribution. This hypothesis was first examined with respect to the FLIP IMU. A complete analysis of each of the sixteen base periods revealed that none of these subsets of data could be described as the Poisson distribution. In most cases, the difference between the computed value of the K-S test statistic (MAX D) and the critical value (D_{crit}) was substantial. To illustrate the general lack of fit, the values of the test statistics for the tests on the Poisson distribution for the FLIP IMU are summarized in Table 1. The fact that the data for each

TABLE 1

VALUES OF THE TEST STATISTIC COMPUTED FOR THE SIMFIT K-S TEST IN THE POISSON DISTRIBUTION ANALYSIS

(D_{crit} = 0.053 for α = 0.10)

Base Period	Computed Value of Test Statistic		
	FLIP	LN-15	KT-73
71-3	0.064	. . .	. . .
71-4	0.070	. . .	. . .
72-1	0.081	. . .	0.222
72-2	0.075	. . .	0.206
72-3	0.076	. . .	0.208
72-4	0.082	. . .	0.227
73-1	0.090	. . .	0.224
73-2	0.103	. . .	0.216
73-3	0.113	. . .	0.216
73-4	0.110	. . .	0.215
74-1	0.120	0.106	0.217
74-2	0.123	0.114	0.225
74-3	0.131	0.121	0.237
74-4	0.139	0.133	0.238
75-1	0.150	0.151	0.245
75-2	0.156	0.157	0.244

base period failed the G-O-F test and that, with only a few exceptions, the failure margin was substantial, indicated that the Poisson distribution could not be accepted as a reasonable descriptor for any of the FLIP failure data.

The test statistics resulting from the SIMFIT analyses of the LN-15 and KT-73 data are also shown in Table 1. As can be readily observed in all cases, the differences between the computed value of the test statistic and the critical value are relatively large, indicating a very poor fit of the Poisson distribution to the base period data. The cumulative result of these thirty-six rejections of the statistical hypotheses (at $\alpha=0.10$) was that the first research hypothesis could be conclusively supported. That is, for these three inertial measurement units, it can be concluded that the daily occurrences of failure do not follow a Poisson distribution.

The Negative Binomial Distribution

At this point in the research, the quest to find a more likely candidate for the underlying failure distribution was begun. The previous SIMFIT products were helpful in this effort. Among the statistics computed in the program for each set of data were the arithmetic mean, the variance, and the variance-to-mean (V/M) ratio.

The V/M ratio was the key to finding a suitable distribution to describe the IMU failure data.

Three theoretical distributions are loosely related through the V/M ratio. These are the Poisson, the binomial, and the negative binomial. The latter two are members of the Bernoulli family of distributions. Considering only these three distributions, for the moment, the following properties can be used as a guide in choosing between them (13:95):

Negative binomial	V/M ratio > 1
Binomial	V/M ratio < 1
Poisson	V/M ratio = 1

Tables 2, 3, and 4 show, by base period, the mean, variance, and V/M ratio of the daily failures for each IMU as computed by SIMFIT.

The V/M ratio for each set of base period data was consistently greater than one. The entire data base was, therefore, analyzed by SIMFIT using the negative binomial as the hypothesized distribution. The results of this effort are shown in Tables 5, 6, and 7 for each IMU by base period. Of the thirty-six data sets examined (sixteen for the FLIP IMU, six for the LN-15 IMU, and fourteen for the KT-73 IMU), twenty-two were successfully fitted with a negative binomial distribution at α=0.10.

For each base period, the SIMFIT program computed the arithmetic mean and variance of data. Using these

TABLE 2

DESCRIPTIVE STATISTICS BY BASE PERIOD
FLIP FAILURE DATA

Base Period	Mean	Variance	V/M Ratio
71-3	0.548	0.755	1.378
71-4	0.566	0.830	1.466
72-1	0.643	1.000	1.555
72-2	0.675	1.014	1.502
72-3	0.688	1.051	1.527
72-4	0.710	1.107	1.559
73-1	0.757	1.190	1.572
73-2	0.791	1.357	1.716
73-3	0.829	1.510	1.822
73-4	0.849	1.603	1.888
74-1	0.839	1.595	1.901
74-2	0.839	1.621	1.932
74-3	0.837	1.656	1.979
74-4	0.834	1.719	2.061
75-1	0.807	1.761	2.182
75-2	0.793	1.788	2.254

TABLE 3

DESCRIPTIVE STATISTICS BY BASE PERIOD
LN-15 FAILURE DATA

Base Period	Mean	Variance	V/M Ratio
74-1	0.692	1.212	1.752
74-2	0.757	1.327	1.753
74-3	0.797	1.404	1.762
74-4	0.864	1.578	1.826
75-1	0.956	1.924	2.012
75-2	1.029	2.076	2.018

TABLE 4

DESCRIPTIVE STATISTICS BY BASE PERIOD
KT-73 FAILURE DATA

Base Period	Mean	Variance	V/M Ratio
72-1	1.378	4.052	2.941
72-2	1.477	4.174	2.826
72-3	1.671	4.670	2.795
72-4	1.758	5.036	2.864
73-1	1.743	4.748	2.724
73-2	1.735	4.580	2.640
73-3	1.647	4.277	2.597
73-4	1.636	4.227	2.584
74-1	1.586	3.940	2.484
74-2	1.595	4.012	2.515
74-3	1.558	3.912	2.511
74-4	1.501	3.744	2.494
75-1	1.501	3.889	2.591
75-2	1.478	3.748	2.536

TABLE 5

NEGATIVE BINOMIAL SIMFIT RESULTS FOR THE FLIP FAILURE DATA BY BASE PERIOD

(D_{crit} = 0.053 for α = 0.10)

Base Period	Max D	K-S Test	Negative Binomial Parameters	
			m	p
71-3	0.084	Failed	1	0.726
71-4	0.044	Passed	1	0.682
72-1	0.037	Passed	1	0.643
72-2	0.081	Failed	1	0.666
72-3	0.076	Failed	1	0.655
72-4	0.068	Failed	1	0.642
73-1	0.076	Failed	1	0.636
73-2	0.026	Passed	1	0.582
73-3	0.018	Passed	1	0.549
73-4	0.026	Passed	1	0.529
74-1	0.025	Passed	1	0.526
74-2	0.037	Passed	1	0.518
74-3	0.059	Failed	1	0.505
74-4	0.088	Failed	1	0.485
75-1	0.138	Failed	1	0.458
75-2	0.164	Failed	1	0.444

TABLE 6

NEGATIVE BINOMIAL SIMFIT RESULTS FOR THE LN-15 FAILURE DATA BY BASE PERIOD

(D_{crit} = 0.053 for α = 0.10)

Base Period	Max D	K-S Test	Negative Binomial Parameters	
			m	p
74-1	0.037	Passed	1	0.571
74-2	0.017	Passed	1	0.571
74-3	0.028	Passed	1	0.568
74-4	0.027	Passed	1	0.548
75-1	0.038	Passed	1	0.497
75-2	0.022	Passed	1	0.495

TABLE 7

NEGATIVE BINOMIAL SIMFIT RESULTS FOR THE
KT-73 FAILURE DATA BY BASE PERIOD

(D_{crit} = 0.053 for α = 0.10)

Base Period	Max D	K-S Test	Negative Binomial Parameters	
			m	p
72-1	0.134	Failed	1	0.340
72-2	0.081	Failed	1	0.354
72-3	0.038	Passed	1	0.358
72-4	0.050	Passed	1	0.349
73-1	0.031	Passed	1	0.367
73-2	0.030	Passed	1	0.379
73-3	0.028	Passed	1	0.385
73-4	0.029	Passed	1	0.387
74-1	0.043	Passed	1	0.403
74-2	0.043	Passed	1	0.398
74-3	0.049	Passed	1	0.398
74-4	0.060	Failed	1	0.401
75-1	0.083	Failed	1	0.386
75-2	0.078	Failed	1	0.394

statistics, the parameters, m and p, of a negative binomial distribution were computed. The theoretical distribution, thus described, was then compared to the data and analyzed using the K-S G-O-F test. The parameters of the distribution used in each test are also shown in the tables cited above.

Complete negative binomial test results, a detailed numerical description of the data, and the associated data histogram for each base period for each IMU, are located in appendixes as follow:

Appendix	IMU
B	FLIP
C	LN-15
D	KT-73

The tables and histogram presented in these appendixes are products of the SIMFIT computer program. Note that some of the tables in Appendix D, the KT-73 analyses, have the tail of the distribution truncated such that the cumulative probability does not reach a value of 1.000. This was done to facilitate the placement of the tables on the page and does not, in any way, affect the test results.

Other Distributions

Having found that the negative binomial distribution adequately described a majority (61.1 percent) of the base period data sets, the research effort was extended to the investigation of other theoretical distributions. Since many of the common statistical distributions are contained in the SIMFIT computer program, it was relatively easy to compare each of them to any given set of data.

Rather than analyzing every data set, which would have required a large amount of computer time, it was decided to sample from the base periods for each IMU population. Any distributions which appeared to closely describe the data could then be investigated further. The base periods for examination were randomly selected for each IMU. Since the FLIP and KT-73 IMUs had a relatively large number of base periods from which selection could be made, three were chosen from each of these populations. Only two base periods were chosen for the LN-15 IMU.

Tables 8, 9, and 10 contain the K-S test results (MAX D values) from these analyses. Note that none of the test statistics except some of those for the negative binomial runs, are less than the critical D value ($D_{crit} = 0.053$). The negative binomial K-S test results which correspond exactly tc those presented earlier, are

TABLE 8

K-S TEST STATISTICS FOR ALL DISTRIBUTIONS AVAILABLE IN SIMFIT FOR THE FLIP FAILURE DATA

($D_{crit} \doteq 0.053$ for $\alpha = 0.10$)

Distribution	Test Statistics		
	FL-72-2	FL-73-2	FL-74-3
Erlang	. . . .	. . . .	. . . .
Weibull	0.412	0.439	0.433
Gamma	0.414	0.441	0.434
Pearson XI	0.220	0.204	0.183
Lognormal	0.296	0.263	0.268
Normal	. . . .	0.127	0.170
Uniform	0.306	0.313	0.345
Beta	. . . .	. . . .	. . . .
Triangular	0.601	0.573	0.561
Poisson	0.075	0.103	0.131
Negative Binomial	0.081	0.026	0.059
Positive Binomial	. . . .	. . . .	. . . .

TABLE 9

K-S TEST STATISTICS FOR ALL DISTRIBUTIONS AVAILABLE IN SIMFIT FOR THE LN-15 FAILURE DATA

(D_{crit} = 0.053 for α = 0.10)

Distribution	Test Statistics	
	LN-74-2	LN-74-4
Erlang	. . . .	. . . .
Weibull	0.414	0.440
Gamma	0.415	0.443
Pearson XI	0.192	0.178
Lognormal	0.298	0.273
Normal	0.142	0.160
Uniform	0.332	0.325
Beta	. . . .	. . . .
Triangular	0.573	0.546
Poisson	0.114	0.133
Negative Binomial	0.017	0.027
Positive Binomial	. . . .	. . . .

TABLE 10

K-S TEST STATISTICS FOR ALL DISTRIBUTIONS AVAILABLE IN SIMFIT FOR THE KT-73 FAILURE DATA

(D_{crit} = 0.053 for α = 0.10)

Distribution	Test Statistics		
	KT-72-2	KT-73-2	KT-75-2
Erlang	0.088	0.097	0.113
Weibull	0.548	0.543	0.498
Gamma	0.556	0.552	0.507
Pearson XI	0.121	0.127	0.124
Lognormal	0.168	0.152	0.206
Normal	0.209	0.289	0.241
Uniform	0.376	0.332	0.348
Beta	. . . .	. . . .	. . . .
Triangular	0.459	0.396	0.414
Poisson	0.206	0.211	0.244
Negative Binomial	0.081	0.030	0.078
Positive Binomial	. . . .	. . . .	. . . .

shown for comparison with the other twelve distributions. When, in the tables, no value is indicated for a particular distribution, the data were out of limits for the theoretical distribution and no SIMFIT analysis could be performed. On the basis of this sample, it was concluded that none of the distributions examined, except the negative binomial, could be expected to describe the failure data for these IMUs. Research in this area was, then, discontinued.

Summary of Findings

This phase of the research addressed the hypothesis that, for each of the inertial measurement units under study, the daily occurrences of failure do not follow a Poisson distribution. In attempting to fit the Poisson to data in sixteen base periods for the FLIP unit, six base periods for the LN-15 unit, and fourteen base periods for the KT-73 unit, not a single case was identified in which that distribution could be accepted as a description of the data. The first research hypothesis is supported by these findings.

The negative binomial distribution was found to fit data in seven of sixteen base periods for the FLIP unit, all six base periods for the LN-15 unit, and nine of fourteen base periods for the KT-73 unit. No other distributions were found to fit data from a sample of

base periods for each IMU. Therefore, according to the criterion established for this research, it was concluded that the negative binomial is the best distribution to describe the daily occurrences of failure for each of the three IMUs.

CHAPTER IV

APPLICATION OF FINDINGS

The second phase of this research considered the use of a negative binomial demand distribution in the existing requirements computation process. The last two research objectives identified in Chapter I were addressed at this point. The first objective was to determine how to modify the D041 computation to incorporate the identified demand distribution. The other was to determine whether the computation would be sensitive to the modification.

This chapter begins with a review of some of the theory behind the Variable Safety Level (VSL) feature of the D041 computation. A discussion of the complexities surrounding incorporation of a negative binomial demand distribution in VSL follows. These complexities precluded performance of a sensitivity analysis to meet the final objective of the research.

Computation of Requirements

The Variable Safety Level (VSL) feature of the D041 requirements computation is based upon the METRIC model.

> METRIC is a model for determining both requirements and distribution of recoverable items in a two-echelon inventory system. The objective of the model is to determine the base and depot stock levels which minimize total expected backorders for a specific set of items and bases subject to an investment constraint . . . . A backorder occurs at a point in time at which there is an unsatisfied demand at base level [17:6].

Backorder Computation. Most aircraft recoverable spare parts are included in a class of items having a high unit cost or a low demand. The optimal inventory policy associated with this class of items is "to place a reorder immediately whenever a demand occurs [11:1]." With this policy, the state[4] of the inventory at a given time is defined by the number of units in resupply. Units that have been reordered (requisitioned from the depot or due in from base maintenance) are considered to be in resupply until delivered to base supply in serviceable condition. When the number of items in resupply exceeds the spare (safety) stock that is in the system, there are backorders (11:2).

In order to determine the level of spare stock that yields the minimum expected backorders, the steady state[5] probabilities for the number of units in resupply

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[4] *State* is variously defined as a specific measureable condition of a system (c:314) or as a specific value for the random variables of a stochastic process (6:582).

[5] The concept of *steady state* comes from queueing theory. "In the steady state, probabilities are independent of time [6:468]." On the other hand, in the transient state, probabilities are dependent upon the initial

must be used. These probabilities are also known as the state probabilities, and can be alternatively expressed as the probabilities of the number of demands in a time interval of specified length. Expected backorders are computed by the formula:

$$B(s) = \sum_{x=s+1}^{\infty} (x-s)h(x)$$

where:

s is the stock level

h(x) is the steady state probabilities of *x* units in resupply.

In VSL, *h(x)* is a negative binomial probability distribution (7).

<u>Logarithmic Poisson Demand</u>. The literature identifies two different demand processes that have negative binomial state probabilities. The more thoroughly documented of the two processes is that in which demand is assumed to occur according to a logarithmic Poisson process. The logarithmic is one of a family of compound

---

conditions. For an introduction to queueing theory, see *Principles of Operations Research for Management*, Chapter 12 (6).

Poisson processes that are generalizations of the simple Poisson process introduced in Chapter I. With a compound Poisson process, *customers* arrive (i.e., place demands), according to a (simple) Poisson process. If each customer can demand more than one unit at a time, the number of *demands* occurring within a specified interval of time has a compound Poisson distribution. If customers are limited to demanding one unit at a time, demands occur according to a simple Poisson distribution (20:4). In either case, the distribution of time between events, batches of demand for the compound process or individual demands for the simple process, is exponential (11:4).

> The logarithmic Poisson distribution is obtained by considering batches of demand where the number of batches [customer arrivals] follows a Poisson process and the number of demands has a logarithmic distribution [21:8].

The RAND literature cites a theorem regarding system performance under the inventory policy for recoverable items:

> . . . if demand is Poisson, then the number of units in resupply in the steady state, x, is also Poisson for any distribution of resupply [times]. The Poisson state probability depends on the mean of the resupply distribution, but not on the distribution itself [11:2].

For a simple Poisson process, with mean *demand* rate (mean number of demands per iterval of time), λ, and mean resupply time, T, the state probabilities are simple Poisson with rate λT (11:3). Similarly, for a compound Poisson

process with *customer arrival rate*, λ, and mean resupply time, T, the state probabilities are compound Poisson with rate λT (11:7). Sherbrooke demonstrated that the state probabilities derived from a logarithmic Poisson demand process can be expressed as a negative binomial distribution (20:12).

Bayesian Inference. In the same discussion, Sherbrooke acknowledged that "there are other non-compound Poisson processes that also yield negative binomial state probabilities [20:12]." One such process is "the probability distribution of demand, where time is broken into intervals of fixed length and demand is Poisson-distributed with period means selected from a gamma distribution [20:12-13]." This statement would be of only passing interest were it not for the concept of Bayesian inference that is prominent in the literature. Four RAND documents, RM-1413 (27), RM-4362-PR (9), RM-4720-PR (10), and RM-5078-PR (21), argue that Bayesian inference can be applied in demand analysis. The discussion that follows describes this application of Bayesian inference.

It is argued that observed mean demand for an item, x, is insufficient to determine the true mean demand for the item, θ. Using the Bayesian approach, "we can increase our knowledge of an item by analyzing the behavior of related items in the supply system [10:6]." If the

specified item is considered to be one of a large, finite group of items (i.e., the recoverable item inventory), each with a true mean demand, Θ, there is a distribution of Θ values for the whole group. The distribution of Θ values becomes the Bayesian prior probability distribution (9:3). Since the true mean demands for the individual items are not known, the distribution of Θ values must be estimated.

Estimation of the prior distribution requires that the form of the distribution be specified and that the distribution's parameters be estimated. The form of the prior distribution is chosen to reflect what is known about the inventory as a whole--that "most items have low demand while a few may have very high demand [9:5]." Therefore the distribution chosen should accommodate any nonnegative real value for Θ and be skewed to the right. The data used to estimate the parameters of the prior distribution were identified as a cross section of observed demand. This set of data includes the total demand observed over a specified time period for each item in the inventory. It is desirable that the estimated distribution have at least two parameters "so that the first two moments of this prior distribution can be estimated from the cross-sectional data [9:5]."

The essence of the Bayesian approach is that:

> . . . the system of inventory items provides the information for a prior distribution common to all items [and this information is combined] with the demand data on a specific item to give a posterior distribution for that item [10:7-8].

Several forms for the prior distribution were proposed in the literature, however, the gamma distribution was specified for the METRIC model. It should be recognized that when the prior distribution is assumed to be gamma and the demand distribution is assumed to be logarithmic Poisson, the posterior probabilities that can be computed are only approximations. However, when the demand distribution is assumed to be simple Poisson, the posterior probabilities can be computed exactly and yield a negative binomial distribution (21:31).

Implications of Research Findings

The fact that the VSL computation incorporates a negative binomial probability distribution to compute expected backorders could be a consequence of one of two lines of reasoning. One accepts the technique of Bayesian inference and assumes the demand distribution to be simple Poisson and the prior distribution to be gamma. The other rejects the technique of Bayesian inference and assumes the demand distribution to be logarithmic Poisson.

The research methodology specifically addressed the assumption that demand for three IMUs follows a simple Poisson distribution. This assumption was not supported by the research. It has been suggested, however, that the fact that the data were aggregated at daily intervals may have masked the existence of a simple Poisson demand process (18).

The assumption of a logarithmic Poisson demand process was not addressed because SIMFIT does not include a logarithmic (or any other compound) Poisson distribution. Although provisions are available for specifying additional distributions, doing so is a major programming and validation effort which could not be completed within the time constraints imposed on the research. However, a comment about this assumption can be made. Sherbrooke established that "any compound Poisson distribution with a positive, discrete compounding distribution has a variance that equals or exceeds its mean [20:5]." It will be recalled that the demand data for all of the base periods have variance-to-mean ratios greater than one (i.e., the variance exceeds the mean). While this fact does not necessarily imply that the demand distributions are compound Poisson, that possibility cannot be dismissed.

The conclusion that the demand distribution for the three IMUs is negative binomial presents a problem.

The literature does not indicate the distribution of the state probabilities associated with negative binomial demand distributions. It was beyond the present capabilities of the authors to identify the appropriate state probability distribution. Therefore, no analysis could be made of the sensitivity of the expected backorder computation to a substitution of distributions.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The conclusions drawn from the research on the failure data for the FLIP, LN-15, and KT-73 inertial measurement units are discussed in this final chapter. Each research hypothesis is addressed in turn. The chapter concludes with several recommendations for future research.

The First Research Hypothesis

The hypothesis first addressed in this research was that: *For each of the inertial measurement units under study, the daily occurrences of failure do not follow a Poisson distribution.* Statistical tests for goodness-of-fit of the Poisson were performed on failure data partitioned by base period. The Poisson distribution could not be fit to any of the base period data sets. These results strongly support this hypothesis.

Additional investigation indicated that the negative binomial distribution adequately described the data in 61.1 percent of the base periods. None of the other distributions available in SIMFIT fit any of the data. In consonance with the criterion stated at the outset of this research effort, it is concluded that the negative

binomial is the best distribution for describing the occurrences of failure for each of the IMUs studied.

The Second Research Hypothesis

The second hypothesis addressed was: *Spares computations are sensitive to the probability distribution that is assumed to describe the underlying failure pattern.* No conclusions can be drawn from this research that either support or refute this hypothesis because a sensitivity analysis could not be performed.

It was reported that the objective of the VSL computation is to determine the base and depot stock levels that minimize total expected backorders, subject to a monetary constraint. The sensitivity of this computation to a change in the demand distribution would be determined by comparing the optimal stock levels and the associated number of expected backorders that result from using different demand distributions. In order to perform the computations, it is necessary to identify the steady state probabilities for the number of items in resupply for the demand distributions under consideration. A review of the literature failed to reveal the state probabilities associated with a negative binomial demand distribution. Without this critical piece of information, the analysis was impossible.

Recommendations

It was concluded that, in the case of three IMUs, it is inappropriate to assume the existence of a simple Poisson demand distribution. Ideally, this research would have identified a modification to the D041 computation, specifically to the VSL segment, that incorporates a more appropriate demand distribution, the negative binomial. Without that modification, and subsequent comparison of its results with those based on the present assumption, no recommendations regarding the D041 can be made. However, the research findings do suggest some avenues for further research.

1. The assumption of a logarithmic Poisson demand distribution should be examined. Since the failure data in all base periods exhibited variances greater than means, one characteristic of this distribution is established. This research would require development of a logarithmic Poisson subroutine for SIMFIT. The type 1 records in the G078C data system identified the base at which each failure occurred, so identifying the distribution of customer arrivals and the distribution of demands per customer should not be difficult.

2. The avenue of research that appears to be most challenging is the identification of the state probability distribution associated with a negative binomial demand distribution. It will be recalled that queueing theory

provided some of the concepts on which METRIC, and later VSL, was based. Budnick et al., suggest that derivation of analytic solutions to queueing problems poses enormous problems to all but the most accomplished of mathematicians. They recommend obtaining the solution by simulation (6:439).

3. Research should be undertaken to determine whether the finding that demand is best described by a negative binomial distribution applies, as well, to other IMUs. That these IMUs are installed on aircraft with such diverse characteristics suggests that the finding may be generic to IMUs rather than attributable to some peculiarity of their operational environments.

4. The G078C system records much more data than was used in this research. One item that can be obtained from the data base is the number of hours the IMU operated before it failed. This type of data could be used in making an actuarial forecast of failures. Since the assumptions of the D041 computation seem to be violated, it may be more reasonable to adopt the actuarial technique, which is currently used by the Air Force only to develop aircraft engine requirements, rather than attempting to modify the D041. The efficacy of this approach, of course, would depend on whether these findings apply across-the-board to all IMUs or only to these three.

APPENDIXES

APPENDIX A

SIMFIT

SIMFIT is a computer program, written in FORTRAN IV, which is used to compare data to theoretical probability distributions. Each distribution available in the program is handled by a separate subroutine which operates independently from the main program and the other distribution subroutines. The following distributions are available in the SIMFIT program (1):

Cumulative
Erlang
Weibull
Gamma
Pearson XI
Lognormal
Normal
Uniform
Beta
Triangular
Poisson
Negative Binomial
Positive Binomial

In addition to the thirteen distributions listed above, the program provides for the inclusion of up to seven subroutines which must be written by the user to specify other theoretical distributions or empirical distributions of his own design.

Goodness-of-Fit Tests

The program employs both the Kolmogorov-Smirnov (K-S) and the Chi-Square (χ^2) tests for goodness-of-fit (G-O-F). The tests are used to determine whether the values of a sample can reasonably be thought to have

come from a population described by the selected theoretical distribution. The SIMFIT user should familiarize himself completely with the mechanics and theory behind each of these G-O-F tests prior to performing any analysis with SIMFIT.

It should be noted that SIMFIT uses each test exactly as it is used in classical nonparametric statistics. The K-S test treats each observation separately and does not cause a loss of information by combining cells which have small numbers of observations. The χ^2 test, on the other hand, combines cells which contain less than five observations and reduces the degrees of freedom used in determining the χ^2 statistic accordingly.

SIMFIT has statistical tables for both G-O-F tests written directly into the program, but only the values for three confidence levels are included: 90, 95, and 99 percent. If the user desires to use a different confidence level, he must change these tables accordingly. This is done by revising the appropriate lines of coding to reflect the desired values (1).

Attractive Features of SIMFIT

Since the program is written in FORTRAN IV, it can be easily extended and modified to fit the specific needs of the user. Care must be taken, however, to investigate fully the impact of a change in SIMFIT on other parts of

the program. For this reason, the user should become thoroughly familiar with the program listing, which is reasonably well documented, before attempting to modify the coding.

The output is formatted in such a manner that it will fit easily into an 8-1/2 by 11 inch format. This feature is especially useful for the thorough documentation of a research effort.

SIMFIT provides an intermediate printout of some of the input variables as well as some statistical information drawn from the data. Items included in the statistics are the mean, standard deviation, variance, variance-to-mean ratio, and the third and fourth moments about the mean. This information is valuable in accurately describing and identifying the data.

As a result of the computer analysis, specific numerical information is printed out for each cell of the data histogram. The user will find this information essential for detailed data analysis.

In addition to the numerical information provided, the program produces a histogram of the data values. For convenience in picturing the shape of the theoretical distibution being investigated, an approximation of the distribution is superimposed onto the data histogram. While the whole picture is not graphically precise, it does

provide the user with a base from which he can make intuitive interpretations of the distribution as it relates to his data.

A separate subroutine is used to read in data to SIMFIT. This enables the researcher to input data from cards, tape, or a time-sharing file with a minimum of data manipulation. The data may be read in either a formatted or free-field mode depending on the desires or needs of the user.

The program incorporates the flexibility for the user to: (1) specify the parameters of the distribution he wishes to test, or (2) allow SIMFIT to estimate distribution parameters from the data. The former option allows the user to examine his data with respect to a specific member of a family of distributions. When the latter option is selected, care must be taken to note that use of the K-S test may not provide reliable results when population parameters are estimated from sample data (12:86). The χ^2 G-O-F test is *always* used as if the parameters of the distribution have been estimated yielding a χ^2 statistic which indicates a loss of one degree of freedom for each distribution parameter.

Recent Modifications to SIMFIT

In January 1977, the RAND Corporation completed a modification to SIMFIT. Several changes were made to

the original program which include the incorporation of two distributions not in earlier versions: the negative and positive binomial distributions. Other modifications were made to facilitate the input of data, format the output in a more usable form, and correct some of the computational problems which had been identified in earlier usage of the program. The result is a flexible, user-oriented, rapid, and relatively easy to use tool for analyzing the distribution of large or small sets of data.

APPENDIX B

NEGATIVE BINOMIAL TEST RESULTS
FOR THE FLIP IMU

FLIP 140 -- BASE PERIOD 71-3

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.548

SAMPLE STD = 0.869
MAX ERROR = 0.084 PROB (0.10, NUM 522) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.726	0.726	335	378.9	0.084	5.0772
2	1.0	0.925	0.199	114	103.9	0.065	0.9839
3	2.0	0.979	0.055	53	28.5	0.018	21.0900
4	3.0	0.994	0.015	15	7.8	0.004	6.6138
5	4.0	0.998	0.004	4	2.1	0.000	
6	5.0	1.000	0.001	1	0.6	0.000	
7	6.0	1.000	0.000	0	0.2	0.000	
8	7.0	1.000	0.000	0	0.0	0.000	
9	8.0	1.000	0.000	0	0.0	0.000	
10	9.0	1.000	0.000	0	0.0	0.000	
11	10.0	1.000	0.000	0	0.0	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 2.710 SUM 33.765

P = 0.726 M = 1

FLIP IMU -- BASE PERIOD 71-3

```
                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
              LEGEND:
                   I - CELL DENSITY
                   . - FITTED TO THE NEG BINOMIAL
                   * - CUMULATIVE DENSITY MATCHES THEORETICAL

      0.        55.83    111.67    167.50    223.33    279.17    335.00    390.83
       *         *         *         *         *         *         *         *
       ...........................................................................
LT MIN *.
   0.  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII       .
  1.0  *.IIIIIIIIIIIIIIIII.II
  2.0  *.IIII.IIII
  3.0  *..I
  4.0  *.
  5.0  *.
  6.0  *.
  7.0  *.
  8.0  *.
  9.0  *.
 10.0  *.
 11.0  *.
 12.0  *.
 13.0  *.
 14.0  *.
GT MAX *.
 15.0  *.
 16.0  *.
 17.0  *.
 18.0  *.
 19.0  *.
 20.0  *.
```

FLIP IMU -- BASE PERIOD 71-4

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.566

SAMPLE STD = 0.911
MAX ERROR = 0.044 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.682	0.682	332	355.1	0.044	1.5053
2	1.0	0.899	0.217	115	113.1	0.041	0.0331
3	2.0	0.968	0.069	53	36.0	0.008	8.0296
4	3.0	0.990	0.022	13	11.5	0.005	0.2065
5	4.0	0.997	0.007	5	3.6	0.002	
6	5.0	0.999	0.002	3	1.2	0.001	
7	6.0	1.000	0.001	0	0.4	0.000	1.5340
8	7.0	1.000	0.000	0	0.1	0.000	
9	8.0	1.000	0.000	0	0.0	0.000	
10	9.0	1.000	0.000	0	0.0	0.000	
11	10.0	1.000	0.000	0	0.0	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 4.600 SUM 11.309

P = 0.682 M = 1

```
FLIP IHU -- BASE PERIOD 71-4

               HISTOGRAM OF DATA AND CURVE APPROXIMATION
          LEGEND:
               I - CELL DENSITY
               . - FITTED TO THE NEG BINOMIAL
               * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.        55.33     110.67    166.00    221.33    276.67    332.00    387.33
           *         *         *         *         *         *         *         *
           ..........................................................................
   LT MIN *.
       0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII    .
      1.0 *.IIIIIIIIIIIIIIIIIII*
      2.0 *.IIIII.III
      3.0 *.I*
      4.0 *.
      5.0 *.
      6.0 *.
      7.0 *.
      8.0 *.
      9.0 *.
     10.0 *.
     11.0 *.
     12.0 *.
     13.0 *.
     14.0 *.
   GT MAX *.
     15.0 *.
     16.0 *.
     17.0 *.
     18.0 *.
     19.0 *.
     20.0 *.
```

FLIP IMU -- BASE PERIOD 72-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.643

SAMPLE STD = 1.000
MAX ERROR = 0.037 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.643	0.643	316	335.3	0.037	1.1064
2	1.0	0.873	0.229	123	119.5	0.030	0.1012
3	2.0	0.955	0.082	51	42.6	0.014	1.6518
4	3.0	0.984	0.029	19	15.2	0.007	0.9551
5	4.0	0.994	0.010	7	5.4	0.004	0.4635
6	5.0	0.998	0.004	5	1.9	0.002	
7	6.0	0.999	0.001	0	0.7	0.001	
8	7.0	1.000	0.000	0	0.2	0.000	
9	8.0	1.000	0.000	0	0.1	0.000	
10	9.0	1.000	0.000	0	0.0	0.000	
11	10.0	1.000	0.000	0	0.0	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	
					CHI SQ 4.600	SUM	4.278

P = 0.643 M = 1

```
FLIP IHU -- BASE PERIOD 72-1

                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
            LEGEND:
                 I - CELL DENSITY
                 . - FITTED TO THE NEG BINOMIAL
                 * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.       52.67    105.33    158.00    210.67    263.33    316.00    368.67
         *         *         *         *         *         *         *         *
         ...........................................................................
  LT MIN *.
      0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII  .
     1.0 *.IIIIIIIIIIIIIIIIIIIIIII.I
     2.0 *.IIIIIIII.I
     3.0 *.I.I
     4.0 *.*
     5.0 *.
     6.0 *.
     7.0 *.
     8.0 *.
     9.0 *.
    10.0 *.
    11.0 *.
    12.0 *.
    13.0 *.
    14.0 *.
  GT MAX *.
    15.0 *.
    16.0 *.
    17.0 *.
    18.0 *.
    19.0 *.
    20.0 *.
```

FLIP 1MU -- BASE PERIOD 72-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.675

SAMPLE STD = 1.007
MAX ERROR = 0.081 PROB (0.10, NUM 520) = 0.054
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.0	0.666	0.666	304	346.3	0.081	5.1622
2	1.0	0.888	0.222	131	115.7	0.052	2.0277
3	2.0	0.963	0.074	52	38.6	0.026	4.6132
4	3.0	0.988	0.025	20	12.9	0.013	3.8919
5	4.0	0.996	0.008	9	4.3	0.004	
6	5.0	0.999	0.003	4	1.4	0.001	9.1233
7	6.0	1.000	0.001	0	0.5	0.000	
8	7.0	1.000	0.000	0	0.2	0.000	
9	8.0	1.000	0.000	0	0.1	0.000	
10	9.0	1.000	0.000	0	0.0	0.000	
11	10.0	1.000	0.000	0	0.0	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	
					CHI SQ 4.600	SUM	24.818

P = 0.666 M = 1

FLIP IMU -- BASE PERIOD 72-2

```
                 HISTOGRAM OF DATA AND CURVE APPPOXIMATION
             LEGEND:
                 I - CELL DENSITY
                 . - FITTED TO THE NEG BINOMIAL
                 * - CUMULATIVE DENSITY MATCHES THEOPETICAL

          0.       50.67     101.33    152.00    202.67    253.33    304.00    354.67
          *         *         *         *         *         *         *         *
          ...........................................................................
   LT MIN *.
       0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII?        .
      1.0 *.IIIIIIIIIIIIIIIIIIIII.III
      2.0 *.IIIIII.III
      3.0 *.I.I
      4.0 *.I
      5.0 *.
      6.0 *.
      7.0 *.
      8.0 *.
      9.0 *.
     10.0 *.
     11.0 *.
     12.0 *.
     13.0 *.
     14.0 *.
   GT MAX *.
     15.0 *.
     16.0 *.
     17.0 *.
     18.0 *.
     19.0 *.
     20.0 *.
```

FLIP IMU -- BASE PERIOD 72-3

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.688

SAMPLE STD = 1.025
MAX ERROR = 0.076 PROB (0.10, NUM 520) = 0.054
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.655	0.655	301	340.6	0.076	4.6018
2	1.0	0.881	0.226	135	117.5	0.042	2.6031
3	2.0	0.959	0.078	48	40.5	0.028	1.3714
4	3.0	0.986	0.027	21	14.0	0.015	3.5146
5	4.0	0.995	0.009	11	4.8	0.003	
6	5.0	0.998	0.003	4	1.7	0.002	11.1526
7	6.0	0.999	0.001	0	0.4	0.001	
8	7.0	1.000	0.000	0	0.2	0.000	
9	8.0	1.000	0.000	0	0.1	0.000	
10	9.0	1.000	0.000	0	0.0	0.000	
11	10.0	1.000	0.000	0	0.0	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	
					CHI SQ	4.600	SUM 23.243

P = 0.655 M = 1

```
CLIP IMU -- BASE PERIOD 72-3

                HISTOGRAM OF DATA AND CURVE APPROXIMATION
           LEGEND:
                I - CELL DENSITY
                . - FITTED TO THE NEG BINOMIAL
                * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.      50.17     100.33    150.50    200.67    250.83    301.00    351.17
         *         *         *         *         *         *         *         *
         ........................................................................
LT MIN *.
   0.   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII       .
   1.0  *.IIIIIIIIIIIIIIIIIIIIIII.III
   2.0  *.IIIIIIII.I
   3.0  *.I.II
   4.0  *.II
   5.0  *.
   6.0  *.
   7.0  *.
   8.0  *.
   9.0  *.
  10.0  *.
  11.0  *.
  12.0  *.
  13.0  *.
  14.0  *.
GT MAX  *.
  15.0  *.
  16.0  *.
  17.0  *.
  18.0  *.
  19.0  *.
  20.0  *.
```

FLIP IHU -- BASE PERIOD 72-4

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.710

SAMPLE STD = 1.052
MAX ERROR = 0.068 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	Y	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.642	0.642	299	334.5	0.068	3.7618
2	1.0	0.872	0.230	133	119.7	0.043	1.4666
3	2.0	0.954	0.082	52	42.9	0.025	1.9434
4	3.0	0.984	0.029	20	15.3	0.016	1.4093
5	4.0	0.994	0.011	12	5.5	0.004	7.6996
6	5.0	0.998	0.004	5	2.0	0.002	
7	6.0	0.999	0.001	0	0.7	0.001	
8	7.0	1.000	0.000	0	0.3	0.000	
9	8.0	1.000	0.000	0	0.1	0.000	
10	9.0	1.000	0.000	0	0.0	0.000	
11	10.0	1.000	0.000	0	0.0	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	
					CHI SQ 4.600	SUM	16.281

P = 0.642 H = 1

```
FLIP IMU -- BASE PERIOD 72-4

                HISTOGRAM OF DATA AND CURVE APPPOXIMATION
            LEGEND:
                 I - CELL DEN  TY
                 . - FITTED  O THE NEG BINOMIAL
                 * - CUMULATIVE DENSITY MATCHES THEORETICAL

        0.        49.83     99.67     149.50    199.33    249.17    299.00    348.83
        *         *         *         *         *         *         *         *
        ...........................................................................
 LT MIN *.
     0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII        .
    1.0 *.IIIIIIIIIIIIIIIIIIIIIIIII.II
    2.0 *.IIIIIIII.II
    3.0 *.II.I
    4.0 *..I
    5.0 *.I
    6.0 *.
    7.0 *.
    8.0 *.
    9.0 *.
   10.0 *.
   11.0 *.
   12.0 *.
   13.0 *.
   14.0 *.
 GT MAX *.
   15.0 *.
   16.0 *.
   17.0 *.
   18.0 *.
   19.0 *.
   20.0 *.
```

FLIP 2MH -- BASE PERIOD 73-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.757

SAMPLE STD = 1.091
MAX ERROR = 0.076 PROB (0.10, NUM 522) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL		PROBABILITY		FREQUENCY		ABSOLUTE	CHI
NO.	X	CUM	PER CELL	ACTUAL	THEORY	ERROR	SQUARE
1	0.	0.636	0.636	292	331.9	0.076	4.7906
2	1.0	0.867	0.232	131	120.9	0.057	0.8477
3	2.0	0.952	0.084	59	44.0	0.028	5.0922
4	3.0	0.982	0.031	19	16.0	0.023	0.5479
5	4.0	0.994	0.011	16	5.8	0.003	17.6711
6	5.0	0.998	0.004	5	2.1	0.002	
7	6.0	0.999	0.001	0	0.8	0.001	
8	7.0	1.000	0.001	0	0.3	0.000	
9	8.0	1.000	0.000	0	0.1	0.000	
10	9.0	1.000	0.000	0	0.0	0.000	
11	10.0	1.000	0.000	0	0.0	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	
				CHI SQ	4.600	SUM	28.949

P = 0.636 M = 1

```
FLIP IMM -- BASE PERIOD 73-1


                  HISTOGRAM OF DATA AND CURVE APPROXIMATION
             LEGEND:
                  I - CELL DENSITY
                  . - FITTED TO THE NEG BINOMIAL
                  * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.        48.67     97.33     146.00    194.67    243.33    292.00    340.67
          *         *         *         *         *         *         *         *
          ...........................................................................
   LT MIN *.
       0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII       .
      1.0 *.IIIIIIIIIIIIIIIIIIIIIIIII.II
      2.0 *.IIIIIIII.III
      3.0 *.II*
      4.0 *..II
      5.0 *.I
      6.0 *.
      7.0 *.
      8.0 *.
      9.0 *.
     10.0 *.
     11.0 *.
     12.0 *.
     13.0 *.
     14.0 *.
   GT MAX *.
     15.0 *.
     16.0 *.
     17.0 *.
     18.0 *.
     19.0 *.
     20.0 *.
```

CLIP IMU -- BASE PERIOD 73-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.791

SAMPLE STD = 1.165
MAX ERROR = 0.026 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.582	0.582	290	303.4	0.026	0.5894
2	1.0	0.826	0.243	131	126.7	0.017	0.1444
3	2.0	0.927	0.102	55	52.9	0.013	0.0807
4	3.0	0.970	0.042	19	22.1	0.019	0.4377
5	4.0	0.987	0.018	18	9.2	0.003	8.3161
6	5.0	0.995	0.007	6	3.9	0.001	
7	6.0	0.998	0.003	2	1.6	0.002	1.1707
8	7.0	0.999	0.001	0	0.7	0.001	
9	8.0	1.000	0.001	0	0.3	0.000	
10	9.0	1.000	0.000	0	0.1	0.000	
11	10.0	1.000	0.000	0	0.0	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 6.250 SUM 10.739

P = 0.582 M = 1

```
FLIP 1/1 -- BASE PERIOD 73-2

                    HISTOGRAM OF DATA AND CURVE APPROXIMATION
               LEGEND:

                    I - CELL DENSITY
                    . - FITTED TO THE NEG BINOMIAL
                    * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.        48.33       96.67      145.00      193.33      241.67      290.00      338.33
          *          *          *          *          *          *          *          *
          ...........................................................................
  LT MIN *.
      0.  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII   .
     1.0  *.IIIIIIIIIIIIIIIIIIIIIIIIII.I
     2.0  *.IIIIIIIIII.I
     3.0  *.III.
     4.0  *..II
     5.0  *.I
     6.0  *.
     7.0  *.
     8.0  *.
     9.0  *.
    10.0  *.
    11.0  *.
    12.0  *.
    13.0  *.
    14.0  *.
  GT MAX  *.
    15.0  *.
    16.0  *.
    17.0  *.
    18.0  *.
    19.0  *.
    20.0  *.
```

FLIP IMJ -- BASE PERIOD 73-3

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.829

SAMPLE STD = 1.229
MAX ERROR = 0.018 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.549	0.549	286	285.9	0.000	0.0001
2	1.0	0.796	0.248	133	129.0	0.008	0.1231
3	2.0	0.908	0.112	52	58.2	0.004	0.6652
4	3.0	0.959	0.050	19	26.3	0.018	2.0147
5	4.0	0.981	0.023	21	11.9	0.000	7.0477
6	5.0	0.992	0.010	7	5.4	0.003	0.5078
7	6.0	0.996	0.005	2	2.4	0.002	
8	7.0	0.998	0.002	1	1.1	0.002	
9	8.0	0.999	0.001	0	0.5	0.001	
10	9.0	1.000	0.000	0	0.2	0.000	
11	10.0	1.000	0.000	0	0.1	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	
					CHI SQ 6.250		SUM 10.359

P = 0.549 M = 1

```
FLIP IMU -- BASE PERIOD 73-3

                HISTOGRAM OF DATA AND CURVE APPROXIMATION
          LEGEND:
               I - CELL DENSITY
               . - FITTED TO THE NEG BINOMIAL
               * - CUMULATIVE DENSITY MATCHES THEORETICAL

     0.        47.67     95.33     143.00    190.67    238.33    286.00    333.67
     *         *         *         *         *         *         *         *
     ...........................................................................
LT MIN *.
    0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII*
   1.0 *.IIIIIIIIIIIIIIIIIIIIIIIIII*
   2.0 *.IIIIIIIIII .
   3.0 *.III .
   4.0 *.I.II
   5.0 *.*
   6.0 *.
   7.0 *.
   8.0 *.
   9.0 *.
  10.0 *.
  11.0 *.
  12.0 *.
  13.0 *.
  14.0 *.
GT MAX *.
  15.0 *.
  16.0 *.
  17.0 *.
  18.0 *.
  19.0 *.
  20.0 *.
```

FLIP 141 -- BASE PERIOD 73-4

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.849

SAMPLE STD = 1.266
MAX ERROR = 0.026 PROB (0.10, NUM 522) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.529	0.529	281	276.4	0.009	0.0767
2	1.0	0.779	0.249	139	130.0	0.026	0.6165
3	2.0	0.896	0.117	51	61.2	0.006	1.6961
4	3.0	0.951	0.055	20	28.8	0.010	2.6832
5	4.0	0.977	0.026	21	13.5	0.004	4.1026
6	5.0	0.989	0.012	6	6.4	0.003	0.0218
7	6.0	0.995	0.006	2	3.0	0.001	
8	7.0	0.998	0.003	1	1.4	0.000	
9	8.0	0.999	0.001	0	0.7	0.001	0.8473
10	9.0	0.999	0.001	1	0.3	0.001	
11	10.0	1.000	0.000	0	0.1	0.000	
12	11.0	1.000	0.000	0	0.1	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 7.780 SUM 10.044

P = 0.529 M = 1

```
FLIP 1M1 -- BASE PERIOD 73-4

                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
           LEGEND:
                 I - CELL DENSITY
                 . - FITTED TO THE NEG BINOMIAL
                 * - CUMULATIVE DENSITY MATCHES THEORETICAL

  0.         46.83      93.67      140.50     187.33     234.17     281.00     327.83
  *          *          *          *          *          *          *          *
  ........................................................................................
IT MIN *.
    0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII*
   1.0 *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.II
   2.0 *.IIIIIIIIIII   .
   3.0 *.IIII .
   4.0 *.I.II
   5.0 *.*
   6.0 *.
   7.0 *.
   8.0 *.
   9.0 *.
  10.0 *.
  11.0 *.
  12.0 *.
  13.0 *.
  14.0 *.
GT MAX *.
  15.0 *.
  16.0 *.
  17.0 *.
  18.0 *.
  19.0 *.
  20.0 *.
```

FLIP IM1 -- BASE PERIOD 74-1

KULMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.839

SAMPLE STD = 1.263
MAX ERROR = 0.025 PROB (0.10, NUM 522) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.526	0.526	288	274.8	0.025	0.6363
2	1.0	0.776	0.249	127	130.1	0.019	0.0756
3	2.0	0.894	0.118	57	61.6	0.010	0.3483
4	3.0	0.950	0.056	20	29.2	0.007	2.8933
5	4.0	0.976	0.026	21	13.8	0.007	3.7243
6	5.0	0.989	0.013	5	6.5	0.004	0.3657
7	6.0	0.995	0.006	2	3.1	0.002	
8	7.0	0.997	0.003	1	1.5	0.001	
9	8.0	0.999	0.001	0	0.7	0.001	0.9744
10	9.0	0.999	0.001	1	0.3	0.001	
11	10.0	1.000	0.000	0	0.2	0.000	
12	11.0	1.000	0.000	0	0.1	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 7.780 SUM 9.018

P = 0.526 M = 1

```
FLIP IMU -- BASE PERIOD 74-1


            HISTOGRAM OF DATA AND CURVE APPROXIMATION
         LEGEND:
            I - CELL DENSITY
            . - FITTED TO THE NEG BINOMIAL
            * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.       48.00      96.00     144.00     192.00     240.00     288.00     336.00
          *          *          *          *          *          *          *          *
          ............................................................................
   LT MIN *.
       0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.III
      1.0 *.IIIIIIIIIIIIIIIIIIIIIIIIII.
      2.0 *.IIIIIIIIIII.
      3.0 *.IIII  .
      4.0 *.I.II
      5.0 *.*
      6.0 *.
      7.0 *.
      8.0 *.
      9.0 *.
     10.0 *.
     11.0 *.
     12.0 *.
     13.0 *.
     14.0 *.
   GT MAX *.
     15.0 *.
     16.0 *.
     17.0 *.
     18.0 *.
     19.0 *.
     20.0 *.
```

FLIP IMU -- BASE PERIOD 74-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.839

SAMPLE STD = 1.273
MAX ERROR = 0.037 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.518	0.518	290	270.7	0.037	1.3746
2	1.0	0.767	0.250	127	130.6	0.030	0.0986
3	2.0	0.880	0.120	55	63.0	0.015	1.0145
4	3.0	0.946	0.058	20	30.4	0.005	3.5510
5	4.0	0.974	0.028	21	14.7	0.007	2.7431
6	5.0	0.987	0.014	6	7.1	0.005	0.1623
7	6.0	0.994	0.007	2	3.4	0.002	
8	7.0	0.997	0.003	1	1.6	0.001	0.8365
9	8.0	0.999	0.002	0	0.8	0.000	
10	9.0	0.999	0.001	1	0.4	0.001	
11	10.0	1.000	0.000	0	0.2	0.000	
12	11.0	1.000	0.000	0	0.1	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	
					CHI SQ 7.780	SUM	9.781

P = 0.518 M = 1

```
FLIP IMU -- BASE PERIOD 74-2

                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
             LEGEND:
                 I - CELL DENSITY
                 . - FITTED TO THE NEG BINOMIAL
                 * - CUMULATIVE DENSITY MATCHES THEORETICAL

        0.        48.33     96.67     145.00    193.33    241.67    290.00    338.33
         *         *         *         *         *         *         *         *
         .......................................................................
 LT MIN *.
     0.  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.III
    1.0  *.IIIIIIIIIIIIIIIIIIIIIIIIIII.
    2.0  *.IIIIIIIIIII .
    3.0  *.IIII .
    4.0  *.II.I
    5.0  *.*
    6.0  *.
    7.0  *.
    8.0  *.
    9.0  *.
   10.0  *.
   11.0  *.
   12.0  *.
   13.0  *.
   14.0  *.
 GT MAX  *.
   15.0  *.
   16.0  *.
   17.0  *.
   18.0  *.
   19.0  *.
```

FLIP IHU -- BASE PERIOD 74-3

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.837

SAMPLE STD = 1.287

MAX ERROR = 0.059 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.505	0.505	295	264.3	0.059	3.5752
2	1.0	0.755	0.250	120	130.7	0.038	0.8815
3	2.0	0.879	0.124	57	64.7	0.024	0.9113
4	3.0	0.940	0.061	19	32.0	0.001	5.2793
5	4.0	0.970	0.030	21	15.8	0.009	1.6889
6	5.0	0.985	0.015	7	7.8	0.007	0.0882
7	6.0	0.993	0.007	2	3.9	0.003	
8	7.0	0.996	0.004	1	1.9	0.002	1.3450
9	8.0	0.998	0.002	0	0.9	0.000	
10	9.0	0.999	0.001	1	0.5	0.001	
11	10.0	1.000	0.000	0	0.2	0.000	
12	11.0	1.000	0.000	0	0.1	0.000	
13	12.0	1.000	0.000	0	0.1	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 7.780 SUM 13.769

P = 0.505 M = 1

FLIP IMM -- BASE PERIOD 74-3

```
                     HISTOGRAM OF DATA AND CURVE APPROXIMATION
                LEGEND:
                     I - CELL DENSITY
                     . - FITTED TO THE NEG BINOMIAL
                     * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.        49.17      98.33     147.50     196.67     245.83     295.00     344.17
         *         *          *         *          *          *          *          *
         ...........................................................................
 LT MIN *.
     0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIII
    1.0 *.IIIIIIIIIIIIIIIIIIIIIIIII .
    2.0 *.IIIIIIIIIIII .
    3.0 *.III  .
    4.0 *.II.I
    5.0 *.*
    6.0 *.
    7.0 *.
    8.0 *.
    9.0 *.
   10.0 *.
   11.0 *.
   12.0 *.
   13.0 *.
   14.0 *.
 GT MAX *.
   15.0 *.
   16.0 *.
   17.0 *.
   18.0 *.
   19.0 *.
   20.0 *.
```

FLIP 1MH -- BASE PERIOD 74-4

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.834

SAMPLE STD = 1.311
MAX ERROR = 0.088 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.485	0.485	300	253.9	0.088	8.3864
2	1.0	0.735	0.250	114	130.6	0.056	2.1191
3	2.0	0.864	0.129	57	67.2	0.037	1.5560
4	3.0	0.930	0.066	22	34.6	0.013	4.5861
5	4.0	0.964	0.034	19	17.8	0.015	0.0804
6	5.0	0.981	0.018	6	9.2	0.009	1.0912
7	6.0	0.990	0.009	2	4.7	0.004	
8	7.0	0.995	0.005	1	2.4	0.001	2.4013
9	8.0	0.997	0.002	1	1.2	0.001	
10	9.0	0.999	0.001	1	0.6	0.001	
11	10.0	0.999	0.001	0	0.3	0.001	
12	11.0	1.000	0.000	0	0.2	0.000	
13	12.0	1.000	0.000	0	0.1	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 7.780 SUM 20.220

P = 0.485 M = 1

FLIP IMU -- BASE PERIOD 74-4

```
                    HISTOGRAM OF DATA AND CURVE APPROXIMATION
               LEGEND:
                    I - CELL DENSITY
                    . - FITTED TO THE NEG BINOMIAL
                    * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.      50.00     100.00    150.00    200.00    250.00    300.00    350.00
          *        *         *         *         *         *         *         *
          ..................................................................................
   LT MIN *.
       0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIII
      1.0 *.IIIIIIIIIIIIIIIIIIIIIII   .
      2.0 *.IIIIIIIIIIII .
      3.0 *.IIII .
      4.0 *.II*
      5.0 *.*
      6.0 *.
      7.0 *
      8.0 *.
      9.0 *.
     10.0 *.
     11.0 *.
     12.0 *.
     13.0 *.
     14.0 *.
   GT MAX *.
     15.0 *.
     16.0 *.
     17.0 *.
     18.0 *.
     19.0 *.
     20.0 *.
```

ELIF 141 -- BASE PERIOD 75-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.807

SAMPLE STD = 1.327
MAX ERROR = 0.138 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.458	0.458	312	239.6	0.138	21.8976
2	1.0	0.706	0.248	108	129.8	0.097	3.6707
3	2.0	0.841	0.135	50	70.4	0.058	5.8911
4	3.0	0.914	0.073	22	38.1	0.027	6.8232
5	4.0	0.953	0.040	18	20.7	0.022	0.3434
6	5.0	0.975	0.021	8	11.2	0.016	0.9134
7	6.0	0.986	0.012	2	6.1	0.008	2.7278
8	7.0	0.993	0.006	1	3.3	0.004	
9	8.0	0.996	0.003	1	1.8	0.002	1.8599
10	9.0	0.998	0.002	1	1.0	0.002	
11	10.0	0.999	0.001	0	0.5	0.001	
12	11.0	0.999	0.001	0	0.3	0.001	
13	12.0	1.000	0.000	0	0.2	0.000	
14	13.0	1.000	0.000	0	0.1	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 9.240 SUM 44.127

P = 0.458 M = 1

FLIP IHI -- BASE PERIOD 75-1

```
              HISTOGRAM OF DATA AND CURVE APPROXIMATION
         LEGEND:
              I - CELL DENSITY
              . - FITTED TO THE NEG BINOMIAL
              * - CUMULATIVE DENSITY MATCHES THEORETICAL

     0.        52.00     104.00     156.00     208.00     260.00     312.00     364.00
     *          *          *          *          *          *          *          *
     ...........................................................................
LT MIN *.
    0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIIIIII
   1.0 *.IIIIIIIIIIIIIIIIIIIII   .
   2.0 *.IIIIIIIIII   .
   3.0 *.IIII  .
   4.0 *.II*
   5.0 *.I.
   6.0 *..
   7.0 *.
   8.0 *.
   9.0 *.
  10.0 *.
  11.0 *.
  12.0 *.
  13.0 *.
  14.0 *.
GT MAX *.
  15.0 *.
  16.0 *.
  17.0 *.
  18.0 *.
  19.0 *.
  20.0 *.
```

FLIP INM -- BASE PERIOD 75-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.793

SAMPLE STD = 1.337
MAX ERROR = 0.164 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	Y	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.444	0.444	318	232.0	0.164	31.8679
2	1.0	0.690	0.247	103	129.1	0.115	5.2721
3	2.0	0.828	0.137	49	71.8	0.071	7.2518
4	3.0	0.904	0.076	24	40.0	0.040	6.3746
5	4.0	0.947	0.043	16	22.2	0.028	1.7475
6	5.0	0.970	0.024	8	12.4	0.020	1.5438
7	6.0	0.983	0.013	0	6.9	0.007	6.8825
8	7.0	0.991	0.007	3	3.8	0.005	
9	8.0	0.995	0.004	1	2.1	0.003	0.6445
10	9.0	0.997	0.002	1	1.2	0.003	
11	10.0	0.998	0.001	0	0.7	0.002	
12	11.0	0.999	0.001	0	0.4	0.001	
13	12.0	1.000	0.000	0	0.2	0.000	
14	13.0	1.000	0.000	0	0.1	0.000	
15	14.0	1.000	0.000	0	0.1	0.000	

CHI SQ 9.240 SUM 61.585

P = 0.444 M = 1

```
FLIF IMN -- BASE PERIOD 75-2


                   HISTOGRAM OF DATA AND CURVE APPROXIMATION
              LEGEND:
                   I - CELL DENSITY
                   . - FITTED TO THE NEG BINOMIAL
                   * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.        53.00     106.00     159.00     212.00     265.00     318.00     371.00
         *         *         *          *          *          *          *          *
         ............................................................................
  LT MIN *.
      0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIIIIIIIII
     1.0 *.IIIIIIIIIIIIIIIIIII    .
     2.0 *.IIIIIIIII    .
     3.0 *.IIII  .
     4.0 *.III.
     5.0 *.I.
     6.0 *..
     7.0 *.
     8.0 *.
     9.0 *.
    10.0 *.
    11.0 *.
    12.0 *.
    13.0 *.
    14.0 *.
  GT MAX *.
    15.0 *.
    16.0 *.
    17.0 *.
    18.0 *.
    19.0 *.
    20.0 *.
```

APPENDIX C

NEGATIVE BINOMIAL TEST RESULTS
FOR THE LN-15 IMU

IN-15 14U -- BASE PERIOD 74-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.692

SAMPLE STD = 1.101
MAX ERROR = 0.037 PROB (0.10, NUM 522) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.571	0.571	317	297.9	0.037	1.2238
2	1.0	0.816	0.245	110	127.9	0.002	2.5027
3	2.0	0.921	0.105	58	54.9	0.008	0.1747
4	3.0	0.966	0.045	24	23.6	0.009	0.0078
5	4.0	0.985	0.019	7	10.1	0.003	0.9611
6	5.0	0.994	0.008	3	4.3	0.001	
7	6.0	0.997	0.004	2	1.9	0.001	0.2353
8	7.0	0.999	0.002	0	0.8	0.001	
9	8.0	1.000	0.001	1	0.3	0.000	
10	9.0	1.000	0.000	0	0.1	0.000	
11	10.0	1.000	0.000	0	0.1	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 6.250 SUM 5.105

P = 0.571 M = 1

LN-15 1HU -- BASE PERIOD 74-1

```
                HISTOGRAM OF DATA AND CURVE APPROXIMATION
           LEGEND:
                I - CELL DENSITY
                . - FITTED TO THE NEG BINOMIAL
                * - CUMULATIVE DENSITY MATCHES THEORETICAL

       0.       52.83     105.67     158.50     211.33     264.17     317.00     369.83
       *         *         *          *          *          *          =          *
       ..........................................................................
 LT MIN *.
     0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.III
    1.0 *.IIIIIIIIIIIIIIIIIIII    .
    2.0 *.IIIIIIIII*
    3.0 *.III*
    4.0 *.*
    5.0 *.
    6.0 *.
    7.0 *.
    8.0 *.
    9.0 *.
   10.0 *.
   11.0 *.
   12.0 *.
   13.0 *.
   14.0 *.
 GT MAX *.
   15.0 *.
   16.0 *.
   17.0 *.
   18.0 *.
   19.0 *.
   20.0 *.
```

LN-15 1/N -- BASE PERIOD 74-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.757

SAMPLE STD = 1.152

MAX ERROR = 0.017 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	Y	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.571	0.571	305	298.6	0.012	0.1351
2	1.0	0.816	0.245	113	128.1	0.017	1.7825
3	2.0	0.921	0.105	60	55.0	0.007	0.4629
4	3.0	0.966	0.045	28	23.6	0.001	0.8307
5	4.0	0.985	0.019	11	10.1	0.003	0.0778
6	5.0	0.994	0.008	3	4.3	0.000	
7	6.0	0.997	0.004	2	1.9	0.001	0.2319
8	7.0	0.999	0.002	0	0.8	0.001	
9	8.0	1.000	0.001	1	0.3	0.000	
10	9.0	1.000	0.000	0	0.1	0.000	
11	10.0	1.000	0.000	0	0.1	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 6.250 SUM 3.521

P = 0.571 K = 1

```
I!-15 I!U -- BASE PERIOD 74-2

              HISTOGRAM OF DATA AND CURVE APPROXIMATION
          LEGEND:
               I - CELL DENSITY
               . - FITTED TO THE NEG BINOMIAL
               * - CUMULATIVE DENSITY MATCHES THEORETICAL

      0.        50.83     101.67    152.50    203.33    254.17    305.00    355.83
      *         *         *         *         *         *         *         *
      ...........................................................................
LT MIN *.
    0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.II
   1.0 *.IIIIIIIIIIIIIIIIIIIIIII  .
   2.0 *.IIIIIIIIII.I
   3.0 *.IIII.I
   4.0 *..I
   5.0 *.
   6.0 *.
   7.0 *.
   8.0 *.
   9.0 *.
  10.0 *.
  11.0 *.
  12.0 *.
  13.0 *.
  14.0 *.
GT MAX *.
  15.0 *.
  16.0 *.
  17.0 *.
  18.0 *.
  19.0 *.
  20.0 *.
```

LN-15 IHU -- BASE PERIOD 74-3

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.797

SAMPLE STD = 1.185
MAX ERROR = 0.028 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.568	0.568	299	297.2	0.004	0.0114
2	1.0	0.814	0.245	112	128.3	0.028	2.0753
3	2.0	0.919	0.106	62	55.4	0.015	0.7837
4	3.0	0.965	0.046	30	23.9	0.003	1.5413
5	4.0	0.985	0.020	14	10.3	0.004	1.3020
6	5.0	0.994	0.009	3	4.5	0.001	
7	6.0	0.997	0.004	2	1.9	0.001	0.3017
8	7.0	0.999	0.002	0	0.8	0.001	
9	8.0	0.999	0.001	1	0.4	0.001	
10	9.0	1.000	0.000	0	0.2	0.000	
11	10.0	1.000	0.000	0	0.1	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	
					CHI SQ	6.250	SUM 6.015

P = 0.568 X = 1

```
EM-15 TMU -- BASE PERIOD 74-3

                HISTOGRAM OF DATA AND CURVE APPROXIMATION
           LEGEND:
                I - CELL DENSITY
                . - FITTED TO THE NEG BINOMIAL
                * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.       49.83     99.67     149.50    199.33    249.17    299.00    348.83
          *         *         *         *         *         *         *         *
          ...........................................................................
  LT MIN *.
      0.  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII*
     1.0  *.IIIIIIIIIIIIIIIIIIIIIII   .
     2.0  *.IIIIIIIIIII.I
     3.0  *.III.II
     4.0  *.I*
     5.0  *.
     6.0  *.
     7.0  *.
     8.0  *.
     9.0  *.
    10.0  *.
    11.0  *.
    12.0  *.
    13.0  *.
    14.0  *.
  GT MAX  *.
    15.0  *.
    16.0  *.
    17.0  *.
    18.0  *.
    19.0  *.
    20.0  *.
```

LH-15 LMU -- BASE PERIOD 74-4

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.864

SAMPLE STD = 1.256
MAX ERROR = 0.027 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.548	0.548	290	286.6	0.007	0.0412
2	1.0	0.796	0.248	112	129.5	0.027	2.3772
3	2.0	0.908	0.112	62	58.6	0.020	0.2013
4	3.0	0.958	0.051	34	26.5	0.006	2.1377
5	4.0	0.981	0.023	18	12.0	0.005	3.0381
6	5.0	0.991	0.010	3	5.4	0.001	1.0744
7	6.0	0.996	0.005	2	2.4	0.000	
8	7.0	0.998	0.002	1	1.1	0.000	
9	8.0	0.999	0.001	1	0.5	0.001	
10	9.0	1.000	0.000	0	0.2	0.000	
11	10.0	1.000	0.000	0	0.1	0.000	
12	11.0	1.000	0.000	0	0.0	0.000	
13	12.0	1.000	0.000	0	0.0	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 6.250 SUM 8.870

P = 0.548 M = 1

```
LN-15 IMU -- BASE PERIOD 74-4


                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
               LEGEND:
                    I - CELL DENSITY
                    . - FITTED TO THE NEG BINOMIAL
                    * - CUMULATIVE DENSITY MATCHES THEORETICAL

        0.        48.33     96.67     145.00    193.33    241.67    290.00    338.33
         *         *         *         *         *         *         *         *
         ...........................................................................
  LT MIN *.
     0.  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII*
    1.0  *.IIIIIIIIIIIIIIIIIIIIIII   .
    2.0  *.IIIIIIIIIII*
    3.0  *.IIII.II
    4.0  *.I.I
    5.0  *..
    6.0  *.
    7.0  *.
    8.0  *.
    9.0  *.
   10.0  *.
   11.0  *.
   12.0  *.
   13.0  *.
   14.0  *.
  GT MAX *.
   15.0  *.
   16.0  *.
   17.0  *.
   18.0  *.
   19.0  *.
   20.0  *.
```

LN-15 IMU -- BASE PERIOD 75-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 0.956

SAMPLE STD = 1.387
MAX ERROR = 0.038 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.497	0.497	280	260.0	0.038	1.5444
2	1.0	0.747	0.250	110	130.7	0.001	3.2917
3	2.0	0.873	0.126	66	65.8	0.001	0.0009
4	3.0	0.936	0.063	37	33.1	0.007	0.4666
5	4.0	0.968	0.032	18	16.6	0.009	0.1123
6	5.0	0.984	0.016	4	8.4	0.001	2.2780
7	6.0	0.992	0.008	4	4.2	0.000	
8	7.0	0.996	0.004	2	2.1	0.000	0.0165
9	8.0	0.998	0.012	1	1.1	0.000	
10	9.0	0.999	0.001	1	0.5	0.001	
11	10.0	0.999	0.001	0	0.3	0.001	
12	11.0	1.000	0.000	0	0.1	0.000	
13	12.0	1.000	0.000	0	0.1	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 7.780 SUM 7.715

P = 0.497 M = 1

```
IH-15 IMU -- BASE PERIOD 75-1

                    HISTOGRAM OF DATA AND CURVE APPROXIMATION
                 LEGEND:
                      I - CELL DENSITY
                      . - FITTED TO THE NEG BINOMIAL
                      * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.       46.67     93.33    140.00    186.67    233.33    280.00    326.67
          *          *         *         *         *         *         *         *
          ...........................................................................
    LT MIN *.
      0.   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIII
     1.0   *.IIIIIIIIIIIIIIIIIIIIIII    .
     2.0   *.IIIIIIIIIIIII*
     3.0   *.IIIIIII*
     4.0   *.II*
     5.0   *..
     6.0   *.
     7.0   *.
     8.0   *.
     9.0   *.
    10.0   *.
    11.0   *.
    12.0   *.
    13.0   *.
    14.0   *.
    GT MAX *.
    15.0   *.
    16.0   *.
    17.0   *.
    18.0   *.
    19.0   *.
    20.0   *.
```

LN-15 IMU -- BASE PERIOD 75-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.029

SAMPLE STD = 1.441

MAX ERROR = 0.022 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.495	0.495	269	258.9	0.019	0.3910
2	1.0	0.745	0.250	109	130.7	0.022	3.6142
3	2.0	0.871	0.126	72	66.0	0.011	0.5437
4	3.0	0.935	0.064	39	33.3	0.000	0.9653
5	4.0	0.967	0.032	19	16.8	0.004	0.2805
6	5.0	0.983	0.016	6	8.5	0.003	0.7333
7	6.0	0.992	0.008	4	4.3	0.001	
8	7.0	0.996	0.004	3	2.2	0.000	0.0459
9	8.0	0.998	0.002	1	1.1	0.000	
10	9.0	0.999	0.001	1	0.6	0.001	
11	10.0	0.999	0.001	0	0.3	0.001	
12	11.0	1.000	0.000	0	0.1	0.000	
13	12.0	1.000	0.000	0	0.1	0.000	
14	13.0	1.000	0.000	0	0.0	0.000	
15	14.0	1.000	0.000	0	0.0	0.000	

CHI SQ 7.780 SUM 6.574

P = 0.495 M = 1

```
IN-15 IMU -- BASE PERIOD 75-2


                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
            LEGEND:
                 I - CELL DENSITY
                 . - FITTED TO THE NEG BINOMIAL
                 * - CUMULATIVE DENSITY MATCHES THEORETICAL

        0.       44.83     89.67    134.50    179.33    224.17    269.00    313.83
         *         *         *         *         *         *         *         *
         ...........................................................................
 LT MIN *.
     0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.III
    1.0 *.IIIIIIIIIIIIIIIIIIIIIIII        .
    2.0 *.IIIIIIIIIIIII.II
    3.0 *.IIIIIII.I
    4.0 *.II.?
    5.0 *.*
    6.0 *.
    7.0 *.
    8.0 *.
    9.0 *.
   10.0 *.
   11.0 *.
   12.0 *.
   13.0 *.
   14.0 *.
 GT MAX *.
   15.0 *.
   16.0 *.
   17.0 *.
   18.0 *.
   19.0 *.
   20.0 *.
```

APPENDIX D

NEGATIVE BINOMIAL TEST RESULTS FOR THE KT-73 IMU

KT-73 IMU -- BASE PERIOD 72-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.378

SAMPLE STD = 2.013
MAX ERROR = 0.134 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.340	0.340	247	177.2	0.134	27.4560
2	1.0	0.565	0.224	107	116.9	0.115	0.8457
3	2.0	0.713	0.148	69	77.2	0.099	0.8631
4	3.0	0.810	0.098	32	50.9	0.063	7.0246
5	4.0	0.875	0.064	21	33.6	0.039	4.7198
6	5.0	0.917	0.043	22	22.2	0.038	0.0012
7	6.0	0.946	0.028	6	14.6	0.022	5.0856
8	7.0	0.964	0.019	7	9.6	0.017	0.7272
9	8.0	0.976	0.012	1	6.4	0.006	4.5235
10	9.0	0.984	0.008	5	4.2	0.008	
11	10.0	0.990	0.005	1	2.8	0.005	0.1356
12	11.0	0.993	0.004	2	1.8	0.005	
13	12.0	0.996	0.002	0	1.2	0.003	
14	13.0	0.997	0.002	1	0.8	0.003	
15	14.0	0.998	0.001	0	0.5	0.002	

CHI SQ 12.020 SUM 51.382

P = 0.348 K = 1

```
YT-73 I4U -- BASE PERIOD 72-1

                HISTOGRAM OF DATA AND CURVE APPROXIMATION
           LEGEND:
                I - CELL DENSITY
                . - FITTED TO THE NEG BINOMIAL
                * - CUMULATIVE DENSITY MATCHES THEORETICAL

        0.       41.17     82.33    123.50    164.67    205.83    247.00    288.17
        *         *         *         *         *         *         *         *
        ..........................................................................
 LT MIN *.
     0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIIIIIIIIII
    1.0 *.IIIIIIIIIIIIIIIIIIIIIIIII  .
    2.0 *.IIIIIIIIIIIIIIII .
    3.0 *.IIIIIII    .
    4.0 *.IIIII  .
    5.0 *.IIII*
    6.0 *.I .
    7.0 *.I.
    8.0 *..
    9.0 *.*
   10.0 *.
   11.0 *.
   12.0 *.
   13.0 *.
   14.0 *.
 GT MAX *.
   15.0 *.
   16.0 *.
   17.0 *.
   18.0 *.
   19.0 *.
   20.0 *.
```

KT-73 IMU -- BASE PERIOD 72-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.477

SAMPLE STD = 2.043
MAX ERROR = 0.081 PROB (0-10, NUM 520) = 0.054
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.354	0.354	226	184.0	0.081	9.5637
2	1.0	0.583	0.229	114	118.9	0.071	0.2024
3	2.0	0.730	0.148	76	76.8	0.070	0.0088
4	3.0	0.826	0.095	33	49.6	0.038	5.5731
5	4.0	0.887	0.062	25	32.1	0.024	1.5567
6	5.0	0.927	0.040	22	20.7	0.027	0.0796
7	6.0	0.953	0.026	6	13.4	0.012	4.0738
8	7.0	0.970	0.017	6	8.6	0.007	0.8103
9	8.0	0.980	0.011	2	5.6	0.000	2.3025
10	9.0	0.987	0.007	6	3.6	0.005	
11	10.0	0.992	0.004	1	2.3	0.002	0.1887
12	11.0	0.995	0.003	2	1.5	0.003	
13	12.0	0.997	0.002	0	1.0	0.001	
14	13.0	0.998	0.001	1	0.6	0.002	
15	14.0	0.999	0.001	0	0.4	0.001	

CHI SQ 12.020 SUM 24.360

P = 0.354 N = 1

```
KT-73 IMU -- BASE PERIOD 72-2

              HISTOGRAM OF DATA AND CURVE APPROXIMATION
          LEGEND:
              I - CELL DENSITY
              . - FITTED TO THE NEG BINOMIAL
              * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.        37.67     75.33     113.00    150.67    188.33    226.00    263.67
          *         *         *         *         *         *         *         *
          ......................................................................................
   LT MIN *.
       0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIIII
      1.0 *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.
      2.0 *.IIIIIIIIIIIIIIIIIIII*
      3.0 *.IIIIIIII    .
      4.0 *.IIIIIII .
      5.0 *.IIII*
      6.0 *.I .
      7.0 *.I.
      8.0 *..
      9.0 *.I
     10.0 *.
     11.0 *.
     12.0 *.
     13.0 *.
     14.0 *.
   GT MAX *.
     15.0 *.
     16.0 *.
     17.0 *.
     18.0 *.
     19.0 *.
     20.0 *.
```

KT-73 140 -- BASE PERIOD 72-3

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.671

SAMPLE STD = 2.161
MAX ERROR = 0.038 PROB (0.10, NUM 520) = 0.054
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.358	0.358	206	186.2	0.038	2.1055
2	1.0	0.588	0.230	111	119.5	0.022	0.6064
3	2.0	0.735	0.148	82	76.7	0.032	0.3621
4	3.0	0.830	0.095	34	49.3	0.003	4.7276
5	4.0	0.891	0.061	32	31.6	0.003	0.0044
6	5.0	0.930	0.039	24	20.3	0.010	0.6728
7	6.0	0.955	0.025	8	13.0	0.001	1.9451
8	7.0	0.971	0.016	9	8.4	0.002	0.0476
9	8.0	0.981	0.010	3	5.4	0.003	1.0479
10	9.0	0.988	0.007	6	3.4	0.002	
11	10.0	0.992	0.004	2	2.2	0.002	0.9635
12	11.0	0.995	0.003	2	1.4	0.003	
13	12.0	0.997	0.002	0	0.9	0.0[illegible]	
14	13.0	0.998	0.001	1	0.6	0.002	
15	14.0	0.999	0.001	0	0.4	0.001	

CHI SQ 12.020 SUM 12.493

P = 0.358 K = 1

```
KT-73 1HU -- BASE PERIOD 72-3

                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
              LEGEND:
                   I - CELL DENSITY
                   . - FITTED TO THE NEG BINOMIAL
                   * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.       34.33     68.67    103.00    137.33    171.67    206.00    240.33
         *         *         *         *         *         *         *         *
         ..........................................................................
  LT MIN *.
      0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIII
     1.0 *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIII .
     2.0 *.IIIIIIIIIIIIIIIIIIIII.I
     3.0 *.IIIIIIIII    .
     4.0 *.IIIIIIII*
     5.0 *.IIII.I
     6.0 *.II.
     7.0 *.I*
     8.0 *..
     9.0 *.*
    10.0 *.
    11.0 *.
    12.0 *.
    13.0 *.
    14.0 *.
  GT MAX *.
    15.0 *.
    16.0 *.
    17.0 *.
    18.0 *.
    19.0 *.
    20.0 *.
```

KT-73 IMU -- BASE PERIOD 72-4

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.758

SAMPLE STD = 2.244
MAX ERROR = 0.050 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	Y	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.349	0.349	208	181.8	0.050	3.7652
2	1.0	0.576	0.227	101	118.4	0.017	2.5496
3	2.0	0.724	0.148	79	77.1	0.021	0.0489
4	3.0	0.820	0.096	37	50.2	0.005	3.4548
5	4.0	0.883	0.063	34	32.7	0.002	0.0553
6	5.0	0.924	0.041	25	21.3	0.005	0.6582
7	6.0	0.950	0.027	12	13.0	0.002	0.2445
8	7.0	0.968	0.017	10	9.0	0.003	0.1089
9	8.0	0.979	0.011	3	5.9	0.002	1.3995
10	9.0	0.985	0.007	6	3.8	0.002	
11	10.0	0.991	0.005	3	2.5	0.003	1.1534
12	11.0	0.994	0.003	2	1.6	0.004	
13	12.0	0.996	0.002	0	1.1	0.002	
14	13.0	0.998	0.001	1	0.7	0.002	
15	14.0	0.998	0.001	0	0.4	0.002	

CHI SQ 12.020 SUM 13.438

P = 0.349 M = 1

KT-73 I-U -- BASE PERIOD 72-4

```
                    HISTOGRAM OF DATA AND CURVE APPROXIMATION
               LEGEND:
                    I - CELL DENSITY
                    . - FITTED TO THE NEG BINOMIAL
                    * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.       34.67     69.33     104.00    138.67    173.33    208.00    242.67
           *         *         *         *         *         *         *         *
           ..........................................................................
  LT MIN *.
      0.   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIII
     1.0   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIII    .
     2.0   *.IIIIIIIIIIIIIIIIIIIIII*
     3.0   *.IIIIIIIIIII   .
     4.0   *.IIIIIIIII*
     5.0   *.IIIII.I
     6.0   *.II*
     7.0   *.I*
     8.0   *..
     9.0   *.*
    10.0   *.
    11.0   *.
    12.0   *.
    13.0   *.
    14.0   *.
  GT MAX   *.
    15.0   *.
    16.0   *.
    17.0   *.
    18.0   *.
    19.0   *.
    20.0   *.
```

KT-73 IMU -- BASE PERIOD 73-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.743

SAMPLE STD = 2.179
MAX ERROR = 0.031 PROB (0.10, NUM 522) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.367	0.367	208	191.7	0.031	1.3917
2	1.0	0.600	0.232	100	121.3	0.010	3.7374
3	2.0	0.747	0.147	77	76.8	0.009	0.0008
4	3.0	0.840	0.093	41	48.6	0.024	1.1806
5	4.0	0.899	0.059	35	30.7	0.013	0.9009
6	5.0	0.936	0.037	26	19.5	0.001	2.2047
7	6.0	0.959	0.024	11	12.3	0.003	0.1393
8	7.0	0.974	0.015	9	7.8	0.001	0.1881
9	8.0	0.984	0.009	3	4.9	0.005	
10	9.0	0.990	0.006	5	3.1	0.001	0.1124
11	10.0	0.993	0.004	3	2.0	0.003	
12	11.0	0.996	0.002	2	1.2	0.004	
13	12.0	0.997	0.002	0	0.6	0.003	
14	13.0	0.998	0.001	0	0.5	0.002	
15	14.0	0.999	0.001	0	0.3	0.001	

CHI SQ 10.640 SUM 9.856

P = 0.367 N = 1

```
KY-73 INU -- BASE PERIOD 73-1

                    HISTOGRAM OF DATA AND CURVE APPROXIMATION
               LEGEND:
                    I - CELL DENSITY
                    . - FITTED TO THE NEG BINOMIAL
                    * - CUMULATIVE DENSITY MATCHES THEORETICAL

        0.       34.67     69.33     104.00    138.67    173.33    208.00    242.67
        *         *         *         *         *         *         *         *
        ...........................................................................
IT MIN  *.
   0.   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIII
   1.0  *.IIIIIIIIIIIIIIIIIIIIIIIIIIII     .
   2.0  *.IIIIIIIIIIIIIIIIIIIII*
   3.0  *.IIIIIIIIIII  .
   4.0  *.IIIIIII.II
   5.0  *.IIII.II
   6.0  *.II*
   7.0  *.I*
   8.0  *..
   9.0  *.I
  10.0  *.
  11.0  *.
  12.0  *.
  13.0  *.
  14.0  *.
GT MAX  *.
  15.0  *.
  16.0  *.
  17.0  *.
  18.0  *.
  19.0  *.
  20.0  *.
```

KT-73 IMU -- BASE PERIOD 73-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.735

SAMPLE STD = 2.140
MAX ERROR = 0.030 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.379	0.379	202	197.4	0.009	0.1075
2	1.0	0.614	0.235	104	122.6	0.027	2.8235
3	2.0	0.760	0.146	78	76.2	0.023	0.0448
4	3.0	0.851	0.091	44	47.3	0.030	0.2304
5	4.0	0.908	0.056	39	29.4	0.011	3.1498
6	5.0	0.943	0.035	21	18.2	0.006	0.4148
7	6.0	0.964	0.022	10	11.3	0.008	0.1572
8	7.0	0.978	0.014	10	7.0	0.003	1.2442
9	8.0	0.986	0.008	3	4.4	0.005	
10	9.0	0.991	0.005	5	2.7	0.001	0.1170
11	10.0	0.995	0.003	3	1.7	0.001	
12	11.0	0.997	0.002	2	1.0	0.003	
13	12.0	0.998	0.001	0	0.7	0.002	
14	13.0	0.999	0.001	0	0.4	0.001	
15	14.0	0.999	0.000	0	0.3	0.001	

CHI SQ 10.640 SUM 8.289

P = 0.379 M = 1

```
KT-73 I4U -- BASE PERIOD 73-2

                    HISTOGRAM OF DATA AND CURVE APPROXIMATION
               LEGEND:
                    I - CELL DENSITY
                    . - FITTED TO THE NEG BINOMIAL
                    * - CUMULATIVE DENSITY MATCHES THEORETICAL

        0.        33.67     67.33     101.00    134.67    168.33    202.00    235.67
         *         *         *         *         *         *         *         *
         ...........................................................................
  LT MIN *.
      0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.II
     1.0 *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIII      .
     2.0 *.IIIIIIIIIIIIIIIIIIIIII.I
     3.0 *.IIIIIIIIIIIII.
     4.0 *.IIIIIIII.III
     5.0 *.IIII.I
     6.0 *.II.
     7.0 *.I*
     8.0 *..
     9.0 *.I
    10.0 *.
    11.0 *.
    12.0 *.
    13.0 *.
    14.0 *.
  GT MAX *.
    15.0 *.
    16.0 *.
    17.0 *.
    18.0 *.
    19.0 *.
    20.0 *.
```

KT-73 IMU -- BASE PERIOD 73-3

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.647

SAMPLE STD = 2.068
MAX ERROR = 0.028 PROB (0.10, NUM 521) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.385	0.385	213	210.7	0.024	0.7538
2	1.0	0.622	0.237	103	1[illegible]3.4	0.016	3.3683
3	2.0	0.768	0.146	71	75.9	0.025	0.3108
4	3.0	0.857	0.090	45	46.6	0.028	0.0573
5	4.0	0.912	0.055	41	28.7	0.004	5.3030
6	5.0	0.946	0.034	18	17.6	0.004	0.0080
7	6.0	0.967	0.021	10	10.8	0.005	0.0645
8	7.0	0.980	0.013	9	6.7	0.001	0.8208
9	8.0	0.987	0.008	3	4.1	0.003	
10	9.0	0.992	0.005	4	2.5	0.000	0.0226
11	10.0	0.995	0.003	3	1.5	0.003	
12	11.0	0.997	0.002	1	1.0	0.003	
13	12.0	0.998	0.001	0	0.6	0.002	
14	13.0	0.999	0.001	0	0.4	0.001	
15	14.0	0.999	0.000	0	0.2	0.001	

CHI SQ 10.640 SUM 10.709

P = 0.385 M = 1

```
KT-73 IMU -- BASE PERIOD 73-3

                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
             LEGEND:
                  I - CELL DENSITY
                  . - FITTED TO THE NEG BINOMIAL
                  * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.       35.50     71.00    106.50    142.00    177.50    213.00    248.50
          *         *         *         *         *         *         *         *
          ...........................................................................
   LT MIN *.
      0.  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.III
     1.0  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIII    .
     2.0  *.IIIIIIIIIIIIIIIIIIII .
     3.0  *.IIIIIIIIIIIII.
     4.0  *.IIIIIII.III
     5.0  *.III.I
     6.0  *.II.
     7.0  *..I
     8.0  *..
     9.0  *.I
    10.0  *.
    11.0  *.
    12.0  *.
    13.0  *.
    14.0  *.
   GT MAX *.
    15.0  *.
    16.0  *.
    17.0  *.
    18.0  *.
    19.0  *.
    20.0  *.
```

KT-73 FMU -- BASE PERIOD 73-4

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.636

SAMPLE STD = 2.056
MAX ERROR = 0.029 PROB (0.10, NUM 522) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	Y	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.387	0.387	214	202.0	0.023	0.7151
2	1.0	0.624	0.237	106	123.8	0.011	2.5666
3	2.0	0.770	0.145	68	75.9	0.026	0.8250
4	3.0	0.859	0.089	45	46.5	0.029	0.0509
5	4.0	0.913	0.055	40	28.5	0.007	4.6096
6	5.0	0.947	0.034	17	17.5	0.008	0.0138
7	6.0	0.967	0.021	14	10.7	0.002	1.0011
8	7.0	0.980	0.013	8	6.6	0.001	0.3092
9	8.0	0.988	0.008	3	4.0	0.001	
10	9.0	0.993	0.005	3	2.5	0.000	0.0386
11	10.0	0.995	0.003	3	1.5	0.003	
12	11.0	0.997	0.002	1	0.9	0.003	
13	12.0	0.998	0.001	0	0.6	0.002	
14	13.0	0.999	0.001	0	0.3	0.001	
15	14.0	0.999	0.000	0	0.2	0.001	

CHI SQ 10.640 SUM 10.130

P = 0.387 M = 1

KT-73 IMU -- BASE PERIOD 73-4

HISTOGRAM OF DATA AND CURVE APPROXIMATION

```
LEGEND:
       I - CELL DENSITY
       . - FITTED TO THE NEG BINOMIAL
       * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.          35.67         71.33         107.00        142.67        178.33        214.00        249.67
          *             *             *             *             *             *             *             *
          ...........................................................................................................
 LT MIN  *.
     0.  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.III
    1.0  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII      .
    2.0  *.IIIIIIIIIIIIIIIIIIIIIIIIII .
    3.0  *.IIIIIIIIIIIIIIIII.
    4.0  *.IIIIIII.IIII
    5.0  *.III*
    6.0  *.II*
    7.0  *..I
    8.0  *..
    9.0  *.
   10.0  *.
   11.0  *.
   12.0  *.
   13.0  *.
   14.0  *.
 GT MAX  *.
   15.0  *.
   16.0  *.
   17.0  *.
   18.0  *.
   19.0  *.
   20.0  *.
```

KI-73 IMU -- BASE PERIOD 74-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.586

SAMPLE STD = 1.985
MAX ERROR = 0.043 PROB (0.10, NUM 522) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.403	0.403	220	210.2	0.019	0.4603
2	1.0	0.643	0.241	104	125.5	0.022	3.6987
3	2.0	0.787	0.144	65	75.0	0.042	1.3337
4	3.0	0.873	0.086	44	44.8	0.043	0.0145
5	4.0	0.924	0.051	43	26.8	0.012	9.8466
6	5.0	0.955	0.031	16	16.0	0.012	0.0000
7	6.0	0.973	0.018	14	9.6	0.004	2.0714
8	7.0	0.984	0.011	9	5.7	0.003	1.9013
9	8.0	0.990	0.007	3	3.4	0.002	
10	9.0	0.994	0.004	1	2.0	0.000	0.3835
11	10.0	0.997	0.002	3	1.2	0.003	
12	11.0	0.998	0.001	0	0.7	0.002	
13	12.0	0.999	0.001	0	0.4	0.001	
14	13.0	0.999	0.000	0	0.3	0.001	
15	14.0	1.000	0.000	0	0.2	0.000	
					CHI SQ 10.640	SUM	19.710

P = 0.403 K = 1

KT-73 IMU -- BASE PERIOD 74-1

HISTOGRAM OF DATA AND CURVE APPPROXIMATION

LEGEND:

I - CELL DENSITY

. - FITTED TO THE NEG BINOMIAL

* - CUMULATIVE DENSITY MATCHES THEORETICAL

```
          0.       36.67     73.33     110.00    146.67    183.33    220.00    256.67
           *         *         *         *         *         *         *         *
           ............................................................................
  LT MIN *.
      0.   *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.III
     1.0   *.IIIIIIIIIIIIIIIIIIIIIIIIIIII     .
     2.0   *.IIIIIIIIIIIIIIIII  .
     3.0   *.IIIIIIIIIII*
     4.0   *.IIIIIII.IIII
     5.0   *.III*
     6.0   *..I
     7.0   *..I
     8.0   *.
     9.0   *.
    10.0   *.
    11.0   *.
    12.0   *.
    13.0   *.
    14.0   *.
  GT MAX   *.
    15.0   *.
    16.0   *.
    17.0   *.
    18.0   *.
    19.0   *.
    20.0   *.
```

KT-73 IMU -- BASE PERIOD 74-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.595

SAMPLE STD = 2.003
MAX ERROR = 0.043 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY SUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.398	0.398	224	207.9	0.031	1.2474
2	1.0	0.637	0.239	183	125.3	0.012	3.9545
3	2.0	0.781	0.144	59	75.5	0.043	3.5927
4	3.0	0.868	0.087	46	45.5	0.042	0.0062
5	4.0	0.921	0.052	40	27.4	0.018	5.8010
6	5.0	0.952	0.032	19	16.5	0.013	0.3773
7	6.0	0.971	0.019	15	9.9	0.004	2.5708
8	7.0	0.983	0.011	12	6.0	0.008	6.0266
9	8.0	0.990	0.007	2	3.6	0.005	
10	9.0	0.994	0.004	0	2.2	0.001	2.4759
11	10.0	0.996	0.003	3	1.3	0.004	
12	11.0	0.998	0.002	0	0.8	0.002	
13	12.0	0.999	0.001	0	0.5	0.001	
14	13.0	0.999	0.001	0	0.3	0.001	
15	14.0	0.999	0.000	0	0.2	0.001	

CHI SQ 10.640 SUM 26.052

P = 0.398 M = 1

```
KT-73 IMU -- BASE PERIOD 74-2

                    HISTOGRAM OF DATA AND CURVE APPROXIMATION
                LEGEND:
                    I - CELL DENSITY
                    . - FITTED TO THE NEG BINOMIAL
                    * - CUMULATIVE DENSITY MATCHES THEORETICAL

           0.         37.33        74.67       112.00      149.33      186.67      224.00      261.33
           *            *            *            *            *            *            *            *
           ...........................................................................................
    LT MIN *.
        0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIII
       1.0 *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII        .
       2.0 *.IIIIIIIIIIIIIIIIIIIIIIII       .
       3.0 *.IIIIIIIII I*
       4.0 *.IIIIII.III
       5.0 *.III.I
       6.0 *.I.II
       7.0 *..II
       8.0 *.
       9.0 *.
      10.0 *.
      11.0 *.
      12.0 *.
      13.0 *.
      14.0 *.
    GT MAX *.
      15.0 *.
      16.0 *.
      17.0 *.
      18.0 *.
      19.0 *.
      20.0 *.
```

KT-73 IMU -- BASE PERIOD 74-3

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.558

SAMPLE STD = 1.978
MAX ERROR = 0.049 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.398	0.398	234	208.2	0.049	3.1864
2	1.0	0.638	0.240	95	125.3	0.009	7.3384
3	2.0	0.782	0.144	55	75.4	0.048	5.5314
4	3.0	0.869	0.087	49	45.4	0.041	0.2865
5	4.0	0.921	0.052	42	27.3	0.013	7.8888
6	5.0	0.952	0.031	18	16.4	0.010	0.1477
7	6.0	0.971	0.019	14	9.9	0.002	1.7027
8	7.0	0.983	0.011	11	6.0	0.008	4.2734
9	8.0	0.990	0.007	3	3.6	0.007	
10	9.0	0.994	0.004	0	2.2	0.002	1.3888
11	10.0	0.996	0.002	2	1.3	0.004	
12	11.0	0.998	0.001	0	0.8	0.002	
13	12.0	0.999	0.001	0	0.5	0.001	
14	13.0	0.999	0.001	0	0.3	0.001	
15	14.0	1.000	0.008	.0	0.2	0.000	

CHI SQ 10.640 SUM 31.664

P = 0.398 M = 1

```
KT-73 IMU -- BASE PERIOD 74-3

                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
            LEGEND:
                 I - CELL DENSITY
                 . - FITTED TO THE NEG BINOMIAL
                 * - CUMULATIVE DENSITY MATCHES THEORETICAL

          0.        39.00      78.00     117.00     156.00     195.00     234.00     273.00
           *          *          *          *          *          *          *          *
           ................................................................................
   LT MIN *.
       0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIII
      1.0 *.IIIIIIIIIIIIIIIIIIIIIIIII      .
      2.0 *.IIIIIIIIIIIIIII     .
      3.0 *.IIIIIIIIII.I
      4.0 *.IIIIII.III
      5.0 *.III*
      6.0 *.I.I
      7.0 *..I
      8.0 *.
      9.0 *.
     10.0 *.
     11.0 *.
     12.0 *.
     13.0 *.
     14.0 *.
   GT MAX *.
     15.0 *.
     16.0 *.
     17.0 *.
     18.0 *.
     19.0 *.
     20.0 *.
```

KT-73 IMU -- BASE PERIOD 74-4

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.501

SAMPLE STD = 1.935
MAX ERROR = 0.060 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.401	0.401	241	209.6	0.060	4.6950
2	1.0	0.641	0.240	94	125.6	0.000	7.9526
3	2.0	0.785	0.144	55	75.3	0.039	5.4542
4	3.0	0.871	0.086	44	45.1	0.041	0.0266
5	4.0	0.923	0.052	45	27.0	0.007	11.9647
6	5.0	0.954	0.031	17	16.2	0.005	0.0405
7	6.0	0.972	0.019	12	9.7	0.001	0.5450
8	7.0	0.983	0.011	10	5.8	0.007	3.0169
9	8.0	0.990	0.007	4	3.5	0.008	
10	9.0	0.994	0.004	0	2.1	0.004	0.4423
11	10.0	0.996	0.002	1	1.3	0.004	
12	11.0	0.998	0.001	0	0.7	0.002	
13	12.0	0.999	0.001	0	0.4	0.001	
14	13.0	0.999	0.001	0	0.3	0.001	
15	14.0	1.000	0.000	0	0.2	0.000	

CHI SQ 10.640 SUM 34.138

P = 0.401 M = 1

KT-73 I4U -- BASE PERIOD 74-4

HISTOGRAM OF DATA AND CURVE APPROXIMATION

```
LEGEND:
     I - CELL DENSITY
     . - FITTED TO THE NEG BINOMIAL
     * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.       40.17     80.33     120.50    160.67    200.83    241.00    281.17
         *         *         *         *         *         *.        *        .*
         ...........................................................................
 LT MIN *.
     0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIII
    1.0 *.IIIIIIIIIIIIIIIIIIIIIII        .
    2.0 *.IIIIIIIIIIIII    .
    3.0 *.IIIIIIIIII.
    4.0 *.IIIII.IIIII
    5.0 *.III*
    6.0 *.I*
    7.0 *..I
    8.0 *.
    9.0 *.
   10.0 *.
   11.0 *.
   12.0 *.
   13.0 *.
   14.0 *.
 GT MAX *.
   15.0 *.
   16.0 *.
   17.0 *.
   18.0 *.
   19.0 *.
   20.0 *.
```

KT-73 IMU -- BASE PERIOD 75-1

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.501

SAMPLE STD = 1.972
MAX ERROR = 0.083 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.386	0.386	245	201.8	0.083	9.2562
2	1.0	0.623	0.237	93	123.9	0.023	7.7200
3	2.0	0.768	0.146	55	76.1	0.017	5.8582
4	3.0	0.858	0.089	39	46.7	0.032	1.2846
5	4.0	0.913	0.055	44	28.7	0.002	8.1394
6	5.0	0.946	0.034	18	17.6	0.002	0.0076
7	6.0	0.967	0.021	12	10.8	0.000	0.1262
8	7.0	0.980	0.013	11	6.7	0.009	2.8417
9	8.0	0.988	0.008	5	4.1	0.011	
10	9.0	0.992	0.005	0	2.5	0.006	0.3858
11	10.0	0.995	0.003	1	1.5	0.005	
12	11.0	0.997	0.002	0	0.9	0.003	
13	12.0	0.998	0.001	0	0.6	0.002	
14	13.0	0.999	0.001	0	0.4	0.001	
15	14.0	0.999	0.000	0	0.2	0.001	

CHI SQ 10.640 SUM 35.619

P = 0.386 M = 1

KT-73 IMU -- BASE PERIOD 75-1

```
                 HISTOGRAM OF DATA AND CURVE APPROXIMATION
            LEGEND:
                 I - CELL DENSITY
                 . - FITTED TO THE NEG BINOMIAL
                 o - CUMULATIVE DENSITY MATCHES THEORETICAL

       0.       40.83     81.67    122.50    163.33    204.17    245.00    285.83
       *          *         *         *   .     .*     . . .*.    .  *  .      *
       ****************************************************************************
LT MIN *,
    0. *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.IIIIIIIIIII
   1.0 *,IIIIIIIIIIIIIIIIIIIIIII        .
   2.0 *,IIIIIIIIIIIIII    .
   3.0 *,IIIIIIIIII .
   4.0 *.IIIIII.III
   5.0 *.III*
   6.0 *,I*
   7.0 *..I
   8.0 *.*
   9.0 *,
  10.0 *.
  11.0 *.
  12.0 *.
  13.0 *.
  14.0 *,
GT MAX *.
  15.0 *,
  16.0 *.
  17.0 *,
  18.0 *,
  19.0 *,
  20.0 *,
```

KT-73 IMU -- BASE PERIOD 75-2

KOLMOGOROV-SMIRNOV ANALYSIS
COMPUTING THE VALUE OF CELLS GENERATED FOR THE
NEG BINOMIAL

SAMPLE AVE = 1.478

SAMPLE STD = 1.936
MAX ERROR = 0.078 PROB (0.10, NUM 523) = 0.053
(IF ERROR IS LE PROB ACCEPT THE DISTRIBUTION)

CELL NO.	X	PROBABILITY CUM	PROBABILITY PER CELL	FREQUENCY ACTUAL	FREQUENCY THEORY	ABSOLUTE ERROR	CHI SQUARE
1	0.	0.394	0.394	247	206.2	0.078	8.0560
2	1.0	0.633	0.239	91	124.9	0.013	9.2062
3	2.0	0.778	0.145	57	75.7	0.023	4.5994
4	3.0	0.865	0.088	39	45.8	0.036	1.0153
5	4.0	0.919	0.053	43	27.8	0.006	8.3781
6	5.0	0.951	0.032	18	16.8	0.004	0.0845
7	6.0	0.970	0.019	12	10.2	0.001	0.3254
8	7.0	0.982	0.012	11	6.2	0.009	3.7904
9	8.0	0.989	0.007	5	3.7	0.011	
10	9.0	0.993	0.004	0	2.3	0.007	0.1655
11	10.0	0.996	0.003	0	1.4	0.004	
12	11.0	0.998	0.002	0	0.8	0.002	
13	12.0	0.999	0.001	0	0.5	0.001	
14	13.0	0.999	0.001	0	0.3	0.001	
15	14.0	0.999	0.000	0	0.2	0.001	

CHI SQ 10.640 SUM 35.621

P = 0.394 M = 1

KT-73 I4U -- BASE PERIOD 75-2

```
                  HISTOGRAM OF DATA AND CURVE APPROXIMATION
             LEGEND:
                  I - CELL DENSITY
                  . - FITTED TO THE NEG BINOMIAL
                  * - CUMULATIVE DENSITY MATCHES THEORETICAL

         0.       41.17     82.33    123.50    164.67    205.83    247.00    288.17
          *         *         *         *         *         *         *         *
          ...........................................................................
 LT MIN  *.
     0.  *.IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII.'IIIIIIIII
    1.0  *.IIIIIIIIIIIIIIIIIIIIII       .
    2.0  *.IIIIIIIIIIIII    .
    3.0  *.IIIIIIIII .
    4.0  *.IIIII.IIII
    5.0  *.III*
    6.0  *.I*
    7.0  *..I
    8.0  *.I
    9.0  *.
   10.0  *.
   11.0  *.
   12.0  *.
   13.0  *.
   14.0  *.
 GT MAX  *.
   15.0  *.
   16.0  *.
   17.0  *.
   18.0  *.
   19.0  *.
   20.0  *.
```

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